

Essays on Multilateral Divisia Monetary Aggregates for Euro Area

By
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Abstract

Achieving the price stability in the economy is the primary objective of the European Central Bank as any other Central Bank. European Central Bank assign a very important role to monetary analysis in its objective of price stability. The role of a good measure of aggregate monetary services across countries in the euro area and the broader European Union is policy relevant. The Divisia monetary aggregation approach is consistent with index number theory and microeconomic aggregation theory, was developed by Barnett (2003, 2007). In the first chapter the multilateral Divisia monetary aggregates are constructed for the union of 24 Euro area countries. The monthly Divisia monetary aggregates for the euro area start from January 2003. In this chapter the currency in circulation, overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months are aggregated. A comparison with the corresponding simple sum monetary aggregates shows that the multilateral Divisia monetary aggregates for the European Monetary Union and European Union is found to perform better and are good indicators of economic trends.

Monetary aggregates have a special role under the "two pillar strategy" of the European Central Bank. Hence, the need for a theoretically consistent measure of monetary aggregates for the European Monetary Union is not trivial. The second chapter analyzes aggregation over monetary assets for the European Monetary Union, and studies the degree of substitutability of the monetary services. The question that is addressed is: "are simple sum aggregates theoretically consistent and an appropriate measure of monetary aggregates for European Monetary Union." The monetary services of the union of eleven European Monetary Union countries is analyzed, which include Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia. The monetary services analyzed are transaction balances (it is a Divisia aggregation of currency in circulation and overnight deposits), deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months. The substitutability among these monetary assets is analyzed for the union of European Monetary Union within the framework of a representative consumer's utility function, using Barnett's (1983) locally flexible functional form, the

minflex Laurent Indirect utility function. The analysis of elasticities with respect to the asset's user-cost prices shows that: (i) transaction balances and deposits with agreed maturity are income elastic and (ii) the monetary assets are not perfect substitutes for each other within the union of European Monetary Union. The necessary condition for the simple sum monetary aggregation is that the component assets are perfect substitutes. Results show that this necessary condition is not satisfied. Hence simple sum aggregation is not theoretically consistent and distorts measurement of the monetary aggregate. In the third chapter, the Divisia monetary aggregates for eleven European Monetary Union countries is used in estimation of nominal GDP of the same union of countries using Markov regime switching model.

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List of Contents

Abstract.....	iii
Acknowledgement.....	v
Chapter 1 Multilateral Divisia Monetary Aggregates for the EMU and EU.....	1
Introduction.....	1
Aggregation within Euro area countries.....	3
Aggregation over Euro area countries.....	4
Data and variables.....	5
Benchmark rate.....	7
European Monetary Union.....	9
European Union.....	11
Conclusion.....	15
Chapter 2 The Demand for Money for EMU: A Flexible Functional Form Approach.....	17
Introduction.....	17
The Consumer's Maximization Problem.....	19
Minflex Laurent Model.....	20
Estimation Procedure.....	21
Estimation Results.....	22
Conclusion.....	33
Chapter 3 Nominal GDP and Divisia M2 Monetary aggregate: In Makov Regime Switching Approach.....	34
Introduction.....	34
Model.....	35
Data and Variables.....	38
Results.....	40
Conclusion.....	44
Appendices.....	45
References.....	57

List of Tables

Table 1.1 Component monetary services of the Divisia aggregates.....	8
Table 1.2 Countries included in the Euro area unions.....	8
Table 2.1 Minflex Laurent model parameter estimates.....	22
Table 2.2 Estimated Income elasticities.....	27
Table 2.3 Estimated price elasticities for the monetary assets.....	28
Table 2.4 Estimated Allen elasticity of substitution for the monetary assets.....	30
Table 2.5 Estimated Morishima elasticity of substitution for the monetary assets.....	31
Table 3.1 Parameter estimates for model without the Divisia M2 monetary aggregate.....	40
Table 3.2 Parameter estimates for model that used the Divisia M2 monetary aggregate.....	41
Table 3.3 Constant transition probabilities for the estimation without the Divisia M2 monetary aggregate.....	41
Table 3.4: Constant transition probabilities for the estimation with the Divisia M2 monetary aggregate.....	41

List of Figures

Figure 1.1 Year-over-year percentage change of EMU-16 M1 monetary aggregate.....	9
Figure 1.2 Year-over-year percentage change for EMU-16 M2 minus monetary aggregate.....	10
Figure 1.3 Year-over-year percentage change for EMU-11 M2 monetary aggregate.....	11
Figure 1.4 Year-over-year percentage change for EU-24 M1 monetary aggregate.....	12
Figure 1.5 Year-over-year percentage change for EU-24 M2 minus monetary aggregate.....	12
Figure 1.6 Year-over-year percentage change for EU-18 M2 monetary aggregate.....	13
Figure 1.7 Year-over-year percentage change for non-financial corporations M1 and M2 minus monetary aggregate for EMU 16 countries.....	14
Figure 1.8 The year-over-year percentage change for non-financial corporations M1 and M2 minus monetary aggregate for EU 24 countries.....	14
Figure 2.1 Year-over-year percentage change of the Divisia and simple sum monetary aggregates for EMU-11 countries.....	54
Figure 2.2 Year-over-year percentage change in Divisia and simple sum monetary aggregate for union of EMU-11.....	56
Figure 2.3 Estimated Income elasticity.....	27

Figure 2.4 Marshallian price elasticity of monetary assets with respect to the user-costs.....	29
Figure 2.5 Allen elasticity of substitution.....	30
Figure 2.6a Morishima Elasticity of Substitution for (i) TB and DAM when the user-cost price of DAM changes; (2) TB and DRN when the user-cost of DRN changes.....	31
Figure 2.6b Morishima Elasticity of Substitution for (i) DAM and TB when the user-cost price of TB changes; (2) DAM and DRN when the user-cost of DRN changes.....	32
Figure 2.6c Morishima Elasticity of Substitution for (i) DRN and TB when the user-cost price of TB changes; (2) DRN and DAM when the user-cost of DAM changes.....	32
Figure 3.1 Nominal GDP for the union of EMU-11 countries.....	39
Figure 3.2 Year-over-year percentage of nominal GDP for the union of EMU-11 countries....	39
Figure 3.3 Markov switching regime probabilities for nominal GDP for model without the Divisia M2 monetary aggregate.....	42
Figure 3.4 Markov switching regime probabilities for nominal GDP for model with the Divisia M2 monetary aggregate.....	43

Chapter 1

Multilateral Divisia Monetary Aggregates for the EMU and EU

Introduction

In the modern economic arrangements we witness the existence of economic unions. In the presence of economic unions like European Union (EU) and closer union European Monetary Union (EMU), there is an interest in studying the performance of these unions and to study the monetary policy impact. In this scenario, the need for a measure of monetary aggregates, which are theoretically consistent for economic unions, such as the EU and EMU, is difficult to obliterate.

With the conception of the European Union, there was an interest in researchers to measure the monetary aggregates in European union, this effort can be categorized into two approaches: direct approach, uses the unilateral representative assumption, which is a very strong and restrictive assumption, which requires convergence of inflation rates and interest rates across countries, implying that the country of residence of a consumer is irrelevant to the unilateral representative agent's decisions Wesche (1997). The alternative indirect approach uses Divisia aggregation within countries and then ad hoc weighting of those within-country indices to aggregate over countries (Reimers (2002); Beyer, Deornik and Hendry (2001), Reimers and Todler (1994). Both the direct approach and indirect approach produces a result that is inconsistent with the monetary and index number theory.

The field of monetary aggregation and index number theory, were first rigorously connected with the literature on microeconomic aggregation and index number theory by Barnett (1980, 1987). This work is based upon the assumption that the data was produced by a single closed economy. In Barnett (2003, 2007), the theory for construction of Divisia monetary aggregates for the Euro area was developed. The theory for

a single country was extended to the multi-country case, for an economic union, both prior to and after the introduction of common currency. A few studies have used Divisia monetary aggregates for the Euro area, such as Stracca (2004) and Darvas (2015), but under restrictive assumptions such as the uniform inflation rate and interest rates across the countries.

In this analysis, multilateral Divisia monetary aggregates are developed for the union of Euro Area countries. The monetary aggregates of 24 countries of Euro area countries is analyzed in this paper, the countries are: Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and UK. These multilateral Divisia monetary aggregate indices are constructed in two steps: firstly, the Divisia monetary aggregates are created for the individual countries of the union; secondly, the Divisia monetary aggregates of the individual countries is aggregated using the expenditure share of the individual country. Hence, the result of this analysis is both the individual country's monetary aggregate indices and the monetary aggregate indices for the union. The results show that multilateral Divisia monetary aggregates are better indicators compared to the corresponding simple sum aggregates. The growth rates of multilateral Divisia monetary aggregates for the unions is lower for the period of recession. They show a divergence from the corresponding simple sum monetary aggregates for the recession period as seen in Barnett and Chauvet (2011) in case of US and Rayton and Pavlyk (2010) in case of UK.

This chapter is organized into various sections, the introduction is followed by the theory of aggregation within Euro area and aggregation over the Euro area countries. The following section enumerates the methodology, Benchmark rate and data sources. The last section shows the Divisia M1 and M2 indices for EMU and EU followed by conclusion.

Euro Area Divisia Monetary Aggregation

The multilateral Divisia monetary aggregation theory developed in Barnett (2003, 2007) is applicable for union at different stages of integration. The theory progresses from a general heterogeneous agents' approach, to a multilateral agent approach, to the most restrictive case of unilateral agent, representing the direction in which the EMU is planned to progress into the future. These results are applicable to the European Union countries and to the European Monetary Union countries.

Aggregation Within Euro Area Countries

Let m_{kji} be the nominal per capita holdings of asset $i \in \{1, 2, \dots, N\}$ located or purchased in country $j \in \{1, 2, \dots, K + Z\}$, where Z is the number of relevant countries that are outside economic union, K be the number of countries in the economic union, and r_{kji} is the holding period after tax yield on asset i located or purchased in country j and owned by an economic agent in country k . Let R_k be the benchmark rate of return in country k , where the benchmark rate of return is received on a pure investment providing no services other than its yield. The real user-cost price of asset i , first derived by Barnett (1978), is

$$\pi_{kji}^*(t) = (R_k(t) - r_{kji}(t)) / (1 + R_k(t)). \quad (1)$$

In this application, the real user-cost price is of asset i located or purchased in country j and owned by residents of country k at time t , while $\pi_{kji} = p_k^* \pi_{kji}^*$ is the corresponding nominal user-cost. The user-cost of a monetary asset measures the foregone interest or opportunity cost of holding monetary asset i , when the higher yielding benchmark asset could have been held.

We define the set $S_k = \{(i, j); m_{kji} > 0 \text{ for all } i, j\}$. Then the real per-capita monetary services aggregate M_k^* and the nominal per-capita monetary services aggregate M_k for each country k are

$$d \log M_k^* = \sum_{(i,j) \in S_k} w_{kji} d \log m_{kji}^* , \quad (2)$$

$$d \log M_k = \sum_{(j,i) \in S_k} w_{kji} d \log m_{kji} . \quad (3)$$

Similarly, the monetary real user-cost price aggregate, Π_k^* , and the monetary nominal user-cost price aggregate, Π_k , are

$$d \log \Pi_k^* = \sum_{(j,i) \in S_k} w_{kji} d \log \pi_{kji}^* , \quad (4)$$

$$d \log \Pi_k = \sum_{(j,i) \in S_k} w_{kji} d \log \pi_{kji} , \quad (5)$$

where the expenditure shares,

$$w_{kji} = \frac{\pi_{kji}^* m_{kji}^*}{\pi_k^* \mathbf{m}_k^*} = \frac{\pi_{kji}^* m_{kji}^*}{\pi_k^* \mathbf{m}_k^*} = \frac{(R_k - r_{kji}) m_{kji}^*}{\sum_{(j,i) \in S_k} (R_k - r_{kji}) m_{kji}^*} = \frac{(R_k - r_{kji}) m_{kji}}{\sum_{(j,i) \in S_k} (R_k - r_{kji}) m_{kji}} \quad \text{and } 0 \leq w_{kji} \leq 1 \text{ for all}$$

$k \in \{1, \dots, K\}$, $j \in \{1, \dots, K + Z\}$, and $i \in \{1, \dots, N\}$. Also it follows that $\sum_{(j,i) \in S_k} w_{kji} = 1$ for all k . In

Appendix A1 displays the year-over-year percentage change of the Divisia aggregate for the 24 Euro Area countries. The Divisia M2 aggregate has component assets of currency in circulation, overnight deposits, deposits with agreed maturity, and deposits redeemable at notice.¹

Aggregation Over Euro Area Countries

The euro area's nominal per-capita monetary service flow, M , and real per-capita monetary service flow, M^* , are given by

$$d \log M = \sum_{k=1}^K W_k d \log (s_k M_k e_k) , \quad (6)$$

¹ Appendix A2 provides the definition of these monetary assets in accordance with the ECB glossary.

$$d \log M^* = \sum_{k=1}^K W_k d \log(s_k M_k^*) . \quad (7)$$

Similarly the euro area's nominal monetary user-cost price, Π , and real monetary user-cost prices, Π^* , are

$$d \log \Pi = \sum_{k=1}^K W_k d \log(\Pi_k e_k) , \quad (8)$$

$$d \log \Pi^* = \sum_{k=1}^K W_k d \log(\Pi_k^*) . \quad (9)$$

where $s_k = H_k / \sum_{k=1}^K H_k$ is country k's share of total economic union population, and H_k is the population of country k. The variable e_k is the exchange rate of country k's currency relative to a market basket of currencies. Country k's expenditure share of the economic union's monetary services flow is

$$\text{given by } W_k = \frac{M_k^* \Pi_k e_k s_k}{\sum_{k=1}^K M_k^* \Pi_k e_k s_k} .$$

The corresponding discrete time Divisia index replaces the differentials $d \log(z_t)$ by finite changes $\log(z_t) - \log(z_{t-1})$ and replaces W_{kt} by $(W_{kt} + W_{k,t-1})/2$. The resulting index is the Törnqvist approximation to the continuous time Divisia index. These results are in per-capita terms

The Data and the Variables

The Euro zone was formed in 1999, but the data for some of the monetary services and their corresponding rates of return are not available until January 2003. Hence, the data for the Euro area countries begin in January 2003. This paper has used monthly data from January 2003 to January 2014. The data for the monetary services, their corresponding rates of return, and the populations and consumer prices of the eleven Euro area countries are acquired from the Statistical Data Warehouse, which is the source provided on the European Central Bank's website. Apart from the European Central Bank, the

central banks of the member countries are also sources of some of our data. Our household data on deposits and interest rates are from the European Central Bank for deposits at Monetary and Financial Institutions. Our currency data are from the central banks of member countries. The data on currency in circulation is taken from the individual countries' central banks, as currency held by banks. The European Central Bank website does not provide the data on currency in circulation. The data for the outstanding amount of overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months are for the households and non-profit institutions serving households. The data for the interest rate for overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months are the Monetary Financial Institutions interest rates for the households and non-profit institutions serving households. Currency is assumed to have a zero own rate of return. The outstanding amount and interest rate data for overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months are from the European Central Bank Data Warehouse.

M1 and M2 Monetary Aggregate

The M1 monetary aggregate contains the most liquid monetary asset components. The European Central Bank has defined the M1 monetary aggregate to include currency in circulation and overnight deposits. Overnight deposits are deposits with next-day maturity and comprises mainly of sight deposits or demand deposits which are fully transferable by check or similar instruments. The European Central Bank definition of the M2 aggregate includes currency in circulation, overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months.

Since January 2003 the data of the Euro zone countries area harmonized according to this definition of overnight deposits, deposits with agreed maturity up to 2 years and deposits redeemable at notice up to 3 months. The database of Belgium has the data for interest rates on saving and demand deposits only until 2003; in case of Greece, the data for sight deposits, saving deposits and time deposits do not have data on interest rates. At the same time France has data for their sight deposits, passbook savings accounts, time

deposits with their interest rates. In light of this diversity in the availability of the data, in this analysis we adopted the European Central Bank definition of the M1 aggregate.

Benchmark Rate

The benchmark rate is the expected rate of return received on a pure investment providing no services other than its yield. In short, the benchmark rate is the rate of return on pure capital. Since it provides no services other than its yield, the benchmark rate must be at least as high as the upper envelope over all the monetary aggregate's component yield-curve-adjusted rates of return. In that upper envelope, we also include the interest rate on loans of maturity of up to one year.

In case of a few countries like Finland, France, and the Netherlands, the interest rate on deposits with agreed maturity of 2 years was greater than the loan rate for a few months. For those periods, 100 basis points were added to the upper envelope to keep the user costs from becoming zero. This procedure is in accordance with Anderson and Jones (2011). In the case of Finland, the interest rate on DAM was higher than the loan rate for two periods of up to one year. For DAM and DRN, those periods were January 2009 to September 2009 and March 2012 to October 2012. For those periods, 0.01 point is added to the loan rate, so that the benchmark rate is highest of all the rates of return on monetary assets. The corresponding periods for France are March 2009 to January 2011 and December 2011 to January 2011. For the Netherlands, the periods are January 2009 to June 2010 and January 2012 to October 2013. In case of EU countries, UK and Lithuania, the interest rate on deposits with agreed maturity of 2 years was greater than the short term loan rate for a few months. For UK, the periods are January 2009 to June 2010, June 2011 to November 2011 and July 2012 to April 2013. For the Lithuania, corresponding period is October 2009 to April 2010.

Table 1.1: The component monetary services of the Divisia aggregates

Divisia Aggregate	Component Monetary Services
Divisia M1	Currency in circulation and Overnight Deposits
Divisia M2 minus	Currency in circulation, Overnight deposits and Deposits with agreed maturity of up to 2 years
Divisia M2	Currency in circulation, Overnight Deposits, Deposits with agreed maturity of up to 2 years and Deposits redeemable up to 3 months.

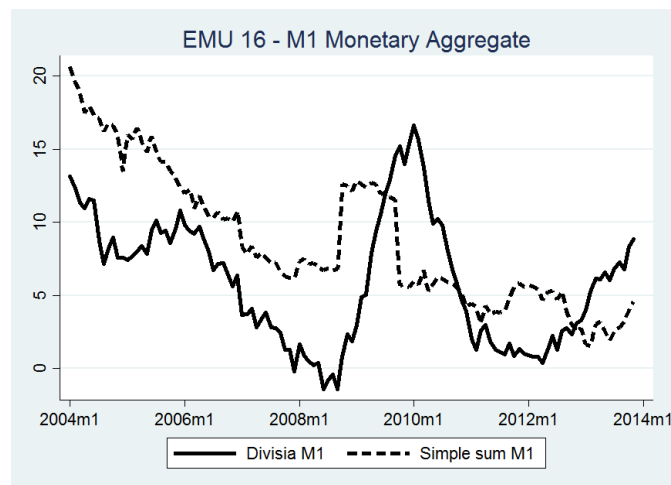
Table 1.2: The countries included in the Euro area unions.

Union	Countries included
EMU-16	Austria, Belgium, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia, Spain
EMU-11	Estonia, Finland, France, Germany, Italy, Luxembourg, Malta, Netherlands, Slovakia, Slovenia
EU-24	Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden, UK
EU-18	Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy,

The European Monetary Union (EMU)

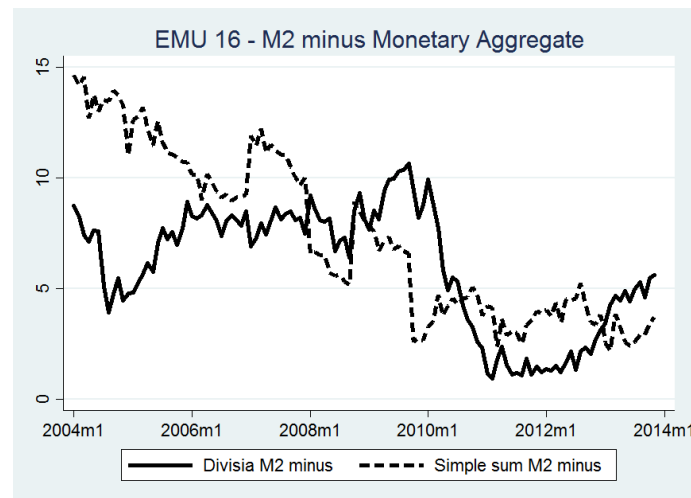
The European Monetary Union (EMU) M1 consists of Currency in circulation and overnight deposits. The data on currency in circulation is taken from the individual countries central banks, the European Central Bank website does not provide the data on the currency in circulation. The data of overnight deposits and their interest rate were taken from the European Central Bank database. The union EMU-16 consists of Austria, Belgium, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia and Spain. In this analysis the Cyprus and Lavia are the European Monetary Union countries that are not include since the data for currency in circulation is not available for these countries. The figure 1.1 shows year-over-year percentage change of EMU-16 M1 monetary aggregate.

Figure 1.1 The year-over-year percentage change of EMU-16 M1 monetary aggregate



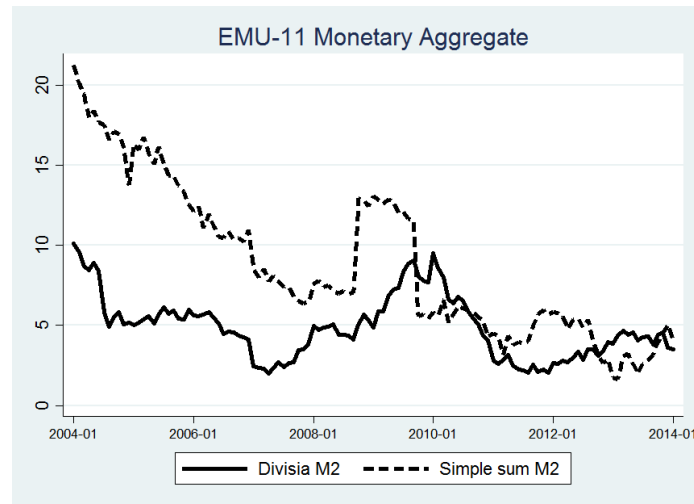
The European Monetary Union Divisia M2 minus aggregate is for EMU-16 union of countries. This excludes Cyprus and Latvia for the aforementioned reason of unavailability of currency in circulation. This Divisia M2 minus aggregate includes currency in circulation, overnight deposits and deposits at agreed maturity. The figure 1.2 shows the year-over-year percentage change for EMU-16 M2 minus monetary aggregate.

Figure 1.2: The year-over-year percentage change for EMU-16 M2 minus monetary aggregate



The EMU-11 Divisia M2 aggregate is for eleven European Monetary Union countries, it excludes Austria, Belgium, Greece, Portugal and Spain from the EMU-16 union of countries. The data on deposits redeemable at notice amount and its interest rate is not available for these countries. This Divisia M2 index aggregates currency in circulation, overnight deposits, deposits with agreed maturity and deposits redeemable at notice. The figure 1.3 shows the year-over-year percentage change for EMU-11 M2 monetary aggregate.

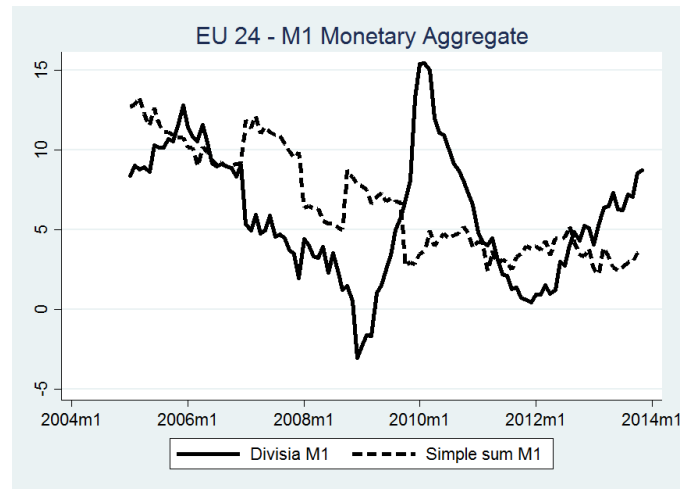
Figure 1.3: The year-over-year percentage change for EMU-11 M2 monetary aggregate



European Union

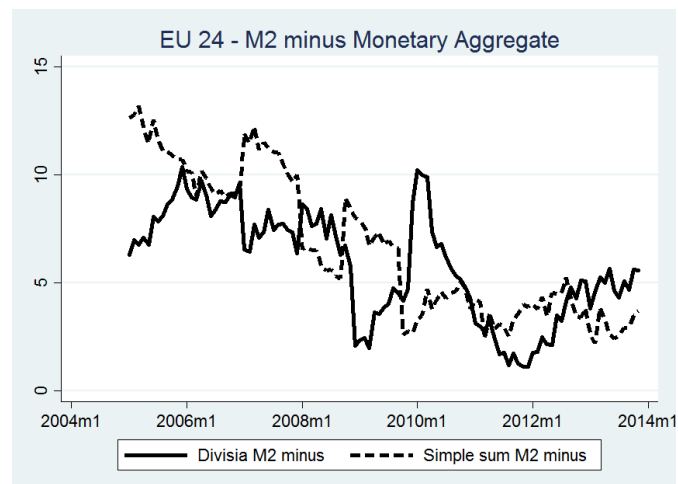
The EU-24 union consists 24 European countries, Austria, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Slovakia, Slovenia, Spain, Sweden and UK. . In this analysis Croatia, Cyprus, Latvia and Romania are the European Union countries that are not include, since the data for currency in circulation is not available for these countries. The index for this group starts from 2004 Jan. The EU-24 Divisia M1 index aggregates currency in circulation and overnight deposits. The figure 1.4 shows the year-over-year percentage change for EU-24 M1 monetary aggregate.

Figure 1.4 The year-over-year percentage change for EU-24 M1 monetary aggregate



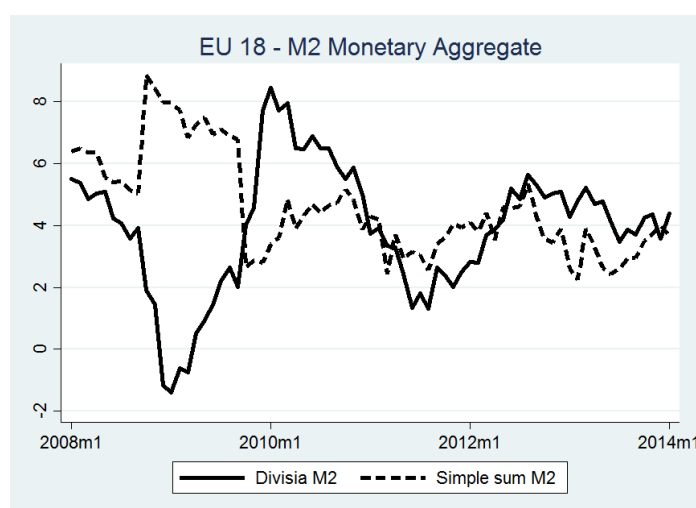
EU-24 Divisia M2 minus monetary aggregate consists of currency in circulation, overnight deposits and deposits with agreed maturity for the 24 European Union countries union. The figure 1.5 shows the year-over-year percentage change for EU-24 M2 minus monetary aggregate.

Figure 1.5: The year-over-year percentage change for EU-24 M2 minus monetary aggregate



EU-18 consists of 18 countries Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, France, Germany, Ireland, Italy, Lithuania, Luxembourg, Malta, Netherlands, Slovakia, Slovenia, Sweden and UK. EU-18 Divisia M2 monetary aggregate includes currency in circulation, overnight deposits, deposits with agreed maturity and deposits redeemable at notice. The figure 1.6 shows the year-over-year percentage change for EU-18 M2 monetary aggregate.

Figure 1.6: The year-over-year percentage change for EU-18 M2 monetary aggregate



Non-Financial Corporations – In this analysis, the Euro area Divisia monetary aggregates for the non-financial corporations are also created. The Monetary Financial Institutions have a different rate of interest for the overnight deposits, deposits with agreed maturity and deposits redeemable at notice for Non-financial corporations. The Divisia M1 and M2 minus indices are created for EMU-16 and EU-24 countries for Non-financial corporations.

Figure 1.7 The year-over-year percentage change for non-financial corporations M1 and M2 minus monetary aggregate for EMU 16 countries.

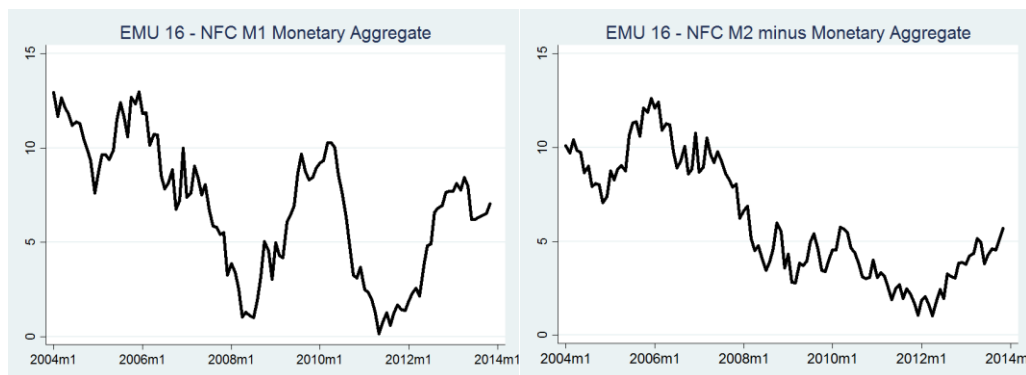
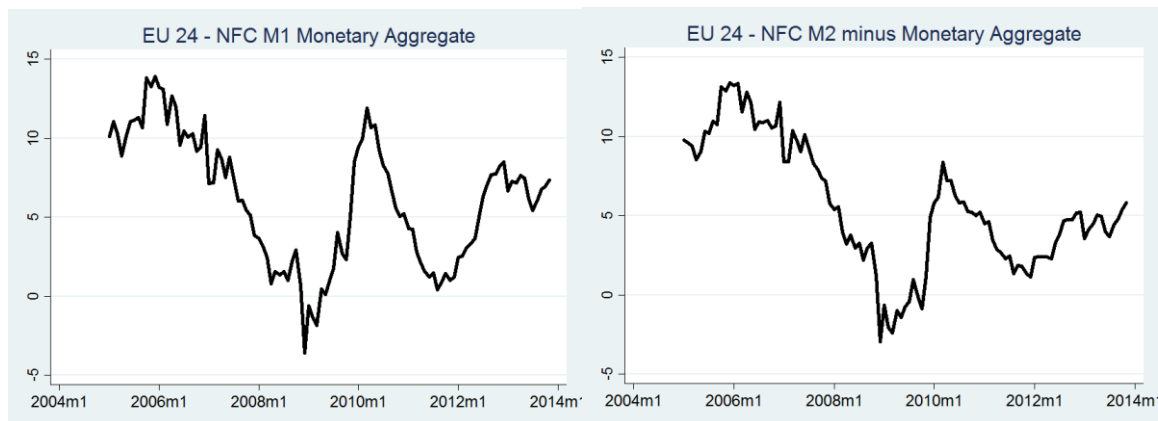


Figure 1.8 The year-over-year percentage change for non-financial corporations M1 and M2 minus monetary aggregate for EU 24 countries.



Individual countries: In the construction of the multilateral Divisia monetary aggregate indices, the individual countries' monetary aggregates are used for weighted aggregation. Hence, the result of this analysis is also the Divisia monetary aggregate indices for the individual countries. A divergence between growth rates of Divisia monetary aggregate and the simple sum monetary aggregate for the period of recession is also evident for individual countries. The growth rates of the Divisia monetary aggregate and the corresponding simple sum monetary aggregate for the 24 Euro area countries is given in Appendix A1.

The year-over-year percentage change in the user-cost aggregate for the EMU and EU union of countries is given in Appendix A3.

Conclusion

This paper examines the monetary services in the 24 European Monetary Union and European Union countries. This is an aggregation over the monetary services of currency in circulation, overnight deposits, and deposits with agreed maturity and deposits redeemable at notice. The results of this analysis are multilateral Divisia monetary aggregates for the union of Euro area countries, with a minimum restrictive assumptions. This makes these indices consistent with index number theory and economic aggregation theory.

These results show that the multilateral Divisia monetary aggregate for EMU and EU countries are a better indicators than the simple sum aggregates and are a good signal of economic trends. When the country Divisia monetary aggregates and the EMU and EU multilateral Divisia monetary aggregate diverge from the simple sum aggregates, the results signaled the recent economic crisis, as observed with US data by Barnett and Chauvet (2011). Barnett and Chauvet (2011) observe that from the 1960s to 2005, the U.S. monetary aggregates and their Divisia counterparts diverge more during periods of high uncertainty than in times of stability. They suggest that this divergence can provide a signal for impending financial instability. For the U.K. Rayton and Pavlyk (2010) demonstrate that the Divisia and simple sum monetary aggregates did not correlate at the start of the recent crisis. During the Great Recession in Germany, Chan and Nautz (2015) found that the information content of the two indices diverged for the recession period. The Divisia monetary aggregate for all the EMU and EU unions show a divergence and these results are consistent with the results from US, UK and Germany. In addition to the divergence of the growth rates of the Divisia monetary aggregates from the corresponding simple sum monetary aggregate, the Divisia monetary aggregates' growth rates were lower than the simple sum monetary aggregates' growth rates prior to the start of the Great Recession. Monetary aggregates are of paramount importance to ECB, since they are part

of the two-pillar strategy employed by ECB for maintaining price stability. The multilateral Divisia monetary aggregates for the Euro area are theoretically consistent and an appropriate measure of monetary services compared to the corresponding simple sum monetary aggregates.

I intend to extend this analysis in two ways: firstly, a comprehensive multilateral Divisia index will be constructed which will be an aggregate of the households and the non-financial corporations for the Euro area. Secondly, an index for inside money for the union of eleven countries of European Monetary Union will be constructed and compare with the data for the corresponding monetary base.

Chapter 2

The Demand for Money for EMU: A Flexible Functional Form Approach

1. Introduction

The European Central Bank (ECB) is among a few central banks that attribute a special role to money under its two pillar strategy. The goal of ECB is to achieve and maintain price level stability in the medium and long-term in the Euro area. ECB achieves this goal by economic analysis and monetary analysis. Monetary analysis, which is one of its pillars, includes analyzing monetary aggregates. Money is found to play a prominent role in Euro area, a long run correlation between money growth and inflation appears to be robust and different policy regimes (Benati 2008,2009). Many studies with European data have confirmed the relationship between monetary growth and inflation. See, Neumann and Greiber (2004); Bruggeman Camba-Mandez, Fischer and Sausa (2005); Gerlach and Assenmacher-Wesche (2005).

The need for a measure of monetary aggregates, which are theoretically consistent for economic unions, such as the EU and EMU, is highly relevant. The field of monetary aggregation and index number theory were first rigorously connected with the literature on microeconomic aggregation and index number theory by Barnett (1980). His initial paper is based on the assumption that the data were produced by a single closed economy. Subsequent studies with those data demonstrated that Divisia monetary aggregates are better measures than simple sum monetary aggregates in terms of policy criteria, such as causality and information content of the aggregate and stability of money demand equations. See, e.g., Barnett, Offenbacher and Spindt (1981, 1984), Belongia and Ireland (2006, 2014, 2015a,b,2016), Serletis and Rahman (2013), and Serletis and Gogas (2014).

In Barnett (2003, 2007), the theory for construction of Divisia monetary aggregates for the Euro area was developed. The theory for a single country was extended to the multi-country case, for an economic union, both prior to and after the introduction of common currency. A few studies have used Divisia monetary aggregates for the Euro area, such as Stracca (2004) and Darvas (2015), but under restrictive assumptions. In this paper, we develop multilateral Divisia monetary aggregates for a group of eleven European Monetary Union (EMU) countries (Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia), following Barnett (2003, 2007). We find that the resulting multilateral Divisia monetary aggregates are more informative than the corresponding simple sum aggregates, the Divisia indices show a lower growth rates for the recession periods which the simple sum aggregates fail to indicate. In appendix B1, figure 2.1 shows the Divisia and the corresponding simple sum monetary aggregates for the eleven European Monetary Union countries and figure 2.2 in appendix B2 shows the Divisia and the corresponding simple sum aggregate for the union of eleven European Monetary Union countries.

A basic question that this analysis answers is: are the simple sum aggregates for the EMU-11 justified? A necessary condition would be that the monetary assets within the EMU-11 are perfectly substitutable. In this paper, the substitutability of the monetary assets within the EMU-11 is investigated using minflex Laurent consumer demand model, a flexible functional form. In the United States, the available results have shown that the monetary assets are not good substitutes. See, e.g., Serletis and Robb (1986). With US data, Serletis and Shahmoradi (2007) have used various functional forms for consumer demand modeling of money demand, including the generalized Leontief, the translog, and the almost Ideal demand system. For the Euro Area, extensive literature exists on the demand for money. Much of that work uses linear combinations of variables, including inflation, output gap, interest rate, and monetary aggregates (e.g., Stracca 2004). A few studies have also included wealth (e.g., Beyer (2008) and Boone et al (2004)).

The chapter proceeds to discuss the minflex Laurent model, our estimation procedure, our results, and conclusions.

2. The Consumer's Maximization Problem

Our assumptions are sufficient for two stage budgeting, as introduced by Strotz (1957, 1959) and Gorman (1959). Hence consumers behave as if they were using sequential expenditure allocation. In the first stage, expenditure allocation is to broad categories. In the second stage, the expenditure allocation is within each broad category.

In the economy, individuals allocate over three types of goods and services: consumption goods, leisure, and the monetary asset services. The services from three enter the representative individual's utility function,

$$u = u(\mathbf{c}, l, \mathbf{m}) \quad (10)$$

where \mathbf{c} is the vector of services of consumption goods, l is leisure, and \mathbf{m} is the vector of services of monetary assets. The consumer maximizes utility subject to the budget constraint, $\mathbf{q}'\mathbf{c} + wl + \mathbf{p}'\mathbf{m} = z$ where \mathbf{q} is the vector of prices of the consumption goods \mathbf{c} , w is the wage rate, \mathbf{p} is a vector of user-cost of the monetary services \mathbf{m} , and z is the quantity of expenditure allocated to the current period in the prior stage intertemporal allocation.

The vector of monetary services is assumed to be weakly separable from consumption goods and leisure.² Hence equation (10) can be written as $u = u(\mathbf{c}, l, f(\mathbf{m}))$ where $f(\mathbf{m})$ is the aggregator function over monetary services. That aggregator function is assumed to be continuous and twice differentiable.

Weak separability in \mathbf{m} requires $\partial(\frac{\partial u}{\partial m_i}) / \partial \xi = 0$ for $\xi = c_i, l$. The consumer's second stage utility

² A substantial literature exists on testing the hypothesis of blockwise weak separability. See, e.g., Hjertstrand, Swofford, and Whitney (2016) and Cherchye, Demuyne, Rock, and Hjerstrand (2015).

maximization problem can be written as $\max_m f(m)$ subject to $\mathbf{p}'\mathbf{m} = y$, where $\mathbf{m} = (m_1, m_2, m_3)$ is the vector of monetary assets, with m_1 = transaction balances, m_2 = deposits with agreed maturity (DAM), and m_3 = deposits redeemable at notice (DRN); $\mathbf{p} = (p_1, p_2, p_3)$ is the corresponding vector of user-costs, and y is the total expenditure on monetary assets allocated during the first stage allocation of z over the three categories of goods and services.

2.1. Minflex Laurent Model

The minflex Laurent model, originated by Barnett (1983), is a special case of the Full Laurent model. Barnett and Lee (1985) showed that among the three flexible functional forms, translog, generalized Leontief, and minflex Laurent, the minflex Laurent model has the largest regular region, and its regular region expands as real income grows. The minflex Laurent model is used to estimate the demand for money for the European data. The full Laurent reciprocal indirect utility function is given by

$$V(\mathbf{v}) = a_0 + 2 \sum_{i=1}^n a_i v_i^{1/2} + \sum_{i=1}^n \sum_{j=1}^n a_{ij} v_i^{1/2} v_j^{1/2} - 2 \sum_{i=1}^n b_i v_i^{-1/2} - \sum_{i=1}^n \sum_{j=1}^n b_{ij} v_i^{-1/2} v_j^{-1/2}, \quad (11)$$

where a_0 , a_i , a_{ij} , b_i , b_{ij} are unknown parameters, and v_i and v_j denote the income normalized prices, p_i / y and p_j / y respectively.

By assuming that $b_i = 0$, $b_{ii} = 0$ for all i , $a_{ij} b_{ij} = 0$ for all i, j , and forcing the off diagonal elements of the symmetric matrices $A \equiv [a_{ij}]$ and $B \equiv [b_{ij}]$ to be nonnegative, equation (11) reduces to the minflex Laurent reciprocal indirect utility function

$$V(\mathbf{v}) = a_0 + 2 \sum_{i=1}^n a_i v_i^{1/2} + \sum_{i=1}^n a_{ii} v_i + \sum_{i=1, i \neq j}^n \sum_{j=1}^n a_{ij}^2 v_i^{1/2} v_j^{1/2} - \sum_{i=1, i \neq j}^n \sum_{j=1}^n b_{ij}^2 v_i^{-1/2} v_j^{-1/2}. \quad (12)$$

By applying Roy's identity to the equations of the indirect utility function of minflex Laurent, the share equations are

$$s_i = \frac{a_i v_i^{1/2} + a_{ii} v_i + \sum_{\substack{j=1 \\ i \neq j}}^n a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{\substack{j=1 \\ i \neq j}}^n b_{ij}^2 v_i^{-1/2} v_j^{-1/2}}{\sum_{i=1}^n a_i v_i^{1/2} + \sum_{i=1}^n a_{ii} v_i + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n b_{ij}^2 v_i^{-1/2} v_j^{-1/2}} . \quad (13)$$

Since the share equations are homogenous of degree zero in the parameters, a normalization is required. Following Barnett and Lee (1985), we impose the following normalization in the estimation of the share equations

$$\sum_{i=1}^n a_{ii} + 2 \sum_{i=1}^n a_i + \sum_{\substack{j=1 \\ i \neq j}}^n a_{ij}^2 - \sum_{\substack{j=1 \\ i \neq j}}^n b_{ij}^2 = 1 . \quad (14)$$

3. Estimation Procedure

The three monetary assets in the consumer utility function are transaction balances (computed as a Divisia aggregate over currency in circulation and overnight deposits), deposits with agreed maturity, and deposits redeemable at notice. The user-costs for these monetary assets are computed using equation 9. To estimate the share equation system, (13), a stochastic version is specified. We assume that the observed share in the i^{th} equation deviates from the true share by an additive term, u_i .

We assume $u \sim N(0, \Omega)$, where $\mathbf{u} = (u_1, \dots, u_n)'$, $\mathbf{0}$ is a null matrix, and $\mathbf{\Omega}$ is the $n \times n$ symmetric positive definite error covariance matrix. The share equations, (13), can be written as

$$\mathbf{s} = \mathbf{g}(\mathbf{v}; \mathbf{\theta}) + \mathbf{u} , \quad (15)$$

where θ is the parameter vector to be estimated. The fact that the budget shares S_i sum to 1, implies that the disturbance covariance matrix is singular. Barten (1969) has shown that full information maximum likelihood estimates of the parameters can be obtained by arbitrarily deleting one equation from the system. The parameters in this paper are estimated following Barten (1969). Estimation is performed using nonlinear full-information maximum likelihood estimation with the TSP (version 5.1) program.

3.1. Estimation Results

Tables 2.1 to 2.5 show the estimated parameters and the elasticity estimates. Table 2.1 shows the parameter estimates of the minflex Laurent model.

Regularity Conditions:

Positivity Condition: The positivity condition is checked by computing the indirect utility function, to confirm $V(\mathbf{v}) > 0$ for all t .

Monotonicity Condition: The monotonicity is checked by computing the gradient vector, to confirm $V'(\mathbf{v}) < 0$ for all t .

Curvature Condition: Curvature is checked by examining negative semidefiniteness of the Allen elasticities of substitution matrix.

Table 2.1

Minflex Laurent model parameter estimates.

Variable	Estimate
a1	0.002406 (0.172985)
a2	-0.003857 (-1.03508)
a3	-0.006280 (-0.88275)

a11	0.28184 (2.42884)
a22	0.259754 (3.96397)
a33	0.473870 (3.72716)
a31	0.00028649 (0.00000159944)
b21	0.00081911 (5.26623)
Loglikelihood	802.568
Positivity violations	0
Monotonicity violations	0
Curvature violations	59
Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01	

The elasticities can be calculated from the estimated budget share equations, which can be written as $x_i = \frac{s_i y}{p_i}$ where $i = 1, 2$ and 3 . The budget share equation can also be written in the logarithmic form as equation 16.

$$\log x_i = \log s_i + \log y - \log p_i \quad (16)$$

The income elasticity is calculated by

$$\eta_{iy} = 1 + \frac{y}{s_i} \frac{\partial s_i}{\partial y} \quad (17)$$

for $i = 1, 2, 3$. The elasticity values in Table 2.2 are evaluated at the mean values of the variables. The values in the parenthesis are t-values (estimate divided by standard error) of the estimates. Table 2.2 shows that the income elasticities for the three monetary assets are positive, so that the monetary assets are normal goods. The transaction balances (TB) and deposits with agreed maturity (DAM) are income elastic, with income elasticities exceeding 1.0, while deposits redeemable at notice (DRN) are income inelastic, with income elasticities less than 1.0. Income elasticity paths over time are shown in Figure 2.3. The elasticity of TB with respect to income is high and does not display much variation, attaining its highest value of 1.182 and lowest value of 1.223. The elasticity of DAM with respect to income displayed its highest value of 1.77 for period April 2009 and lowest value of 1.179 for January 2004. The elasticity of DRN with respect to income is uniformly low, attaining its highest value of 0.64 for period January 2004 and lowest value, 0.285, for April 2009.

The Marshallian (uncompensated) price elasticities are calculated from

$$\eta_{ij} = \left(\frac{p_j}{s_i}\right)\left(\frac{\partial s_i}{\partial p_j}\right) - \delta_{ij} , \quad (18)$$

where $i = 1, 2, 3$; and δ_{ij} is the Kronecker delta, so that $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. The Marshallian price elasticity in equation (18) is derived from the differentiation of equation (16) with respect to prices. The Marshallian price elasticity values in Table 2.3 are evaluated at the mean values of the variables. The values in the parenthesis are t-values (estimate divided by standard error) of the estimates. All own price elasticities are negative, ruling out Giffen goods. All cross-price elasticities are negative, hence all of the assets are found to be gross complements. All price elasticities are less than 1.0 in absolute value, so all of the monetary assets are price inelastic. The income and price elasticities are consistent with the results in Serletis and Robb (1986) in case of US.

The Marshallian price elasticity of TB with respect to the user-costs over time is shown in Figure 2.4, top left graph. The own-price elasticity of TB is low, although it increased slightly in January 2009 to -0.48. The elasticity of TB with respect to the user-cost of DAM is comparatively high and does not show much variation, although it fell slightly in January 2009 to -0.18. The elasticity of TB with respect to the user-cost of DRN showed high variation, with highest value, -0.28, in January 2014 and lowest value, -0.57, in January 2009. The Marshallian price elasticity of DAM with respect to the user-costs over time is shown in Figure 2.4, top right graph. The own-price elasticity of DAM is low, with its highest value, -0.37 in January 2009 and lowest value, -0.58 in May 2009. The elasticity of DAM with respect to the user-costs of TB is comparatively high, with its lowest value, -0.34, in July 2003 and its highest value, -0.095, in April 2009. The elasticity of DAM with respect to the user-costs of DRN displays high variation, increasing to -0.07 in May 2009 and decreasing to -0.46 in July 2003. The Marshallian price elasticity of DRN with respect to the user-costs over time is shown in Figure 2.4, bottom left. The own-price elasticity of DRN is low but very volatile, with a sharp increase to -0.26 in January 2009 and a decrease to -0.55 in October 2013. The elasticity of DRN with respect to the user-cost of TB is less volatile, with its highest value, -0.33 in October 2003 and its lowest value, -0.48 in April 2009. The elasticity of DRN with respect to the user-cost of the DAM is comparatively high and less volatile, with its highest value, -0.105 in May 2009 and its lowest value, -0.25 in November 2008. The income and Marshallian price elasticities over time can be instruments for the European Central Bank and the Monetary and Financial Institutions for regulation of the interest rates to achieve price stability.

The Allen elasticity of substitution is given in Table 2.4, the elasticity values are evaluated at the mean values of the variables. The values in the parenthesis are t-values (estimate divided by standard error) of the estimates. The own-Allen elasticities of substitution are negative, as is consistent with theory. The Allen cross elasticities are positive and less than 1.0 indicating that monetary assets are weak substitutes, but far from perfect substitutes. Allen elasticities of substitution over time are shown in Figure 2.5. The cross elasticities of substitution are positive and less than 1.0, indicating that the monetary assets

are weak substitutes but far from perfect substitutes. Substitutability shows sharp decrease in 2009 and 2013.

Blackorby and Russell (1989) have shown that cross Allen elasticity of substitution may provide ambiguous information and suggest that Morishima elasticity of substitution may be a better measure of substitutability. Blackorby and Russell (1989) and Serletis and Shahmoradi (2005) advocate computing Morishima elasticities using the equation $\sigma_{ij}^m(p, y) = s_i(p, y)(\sigma_{ji}^a(p, y) - \sigma_{ii}^a(p, y))$, where $\sigma_{ji}^a(p, y)$ and $\sigma_{ii}^a(p, y)$ are Allen elasticity of substitution. The Morishima elasticity measures the net change in the compensated demand for good j , when the price of good i changes. Goods will be Morishima complements (substitutes) if an increase in the price of i causes x_i/x_j to decrease (increase). The Morishima elasticity of substitution is shown in Table 2.5. The Morishima elasticity of substitution for all the three monetary services being less than 1.0 shows that all three monetary services are weak substitutes and far from perfect substitutes. Morishima elasticity of substitution of the monetary assets over time is shown in Figure 2.6a, Figure 2.6b and Figure 2.6c. The Morishima elasticity of substitution of the monetary assets over time is less than unity and it shows steep variation in 2008.

Both Allen elasticity of substitution and Morishima elasticity of substitution show that the monetary services for a union of eleven countries of European Monetary Union are not perfect substitutes. This does not satisfy the necessary condition for the simple sum aggregates that components should be perfect substitutes. Hence the simple sum monetary aggregates for this union will be theoretically inconsistent.

Table 2.2

Estimated Income elasticities

Monetary Asset	Income Elasticity
Transaction balances	1.21652 (26.8955)
Deposits with agreed maturity	1.34478 (9.08806)
Deposits redeemable at notice	0.536689 (11.0111)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Figure 2.3

Estimated Income elasticity

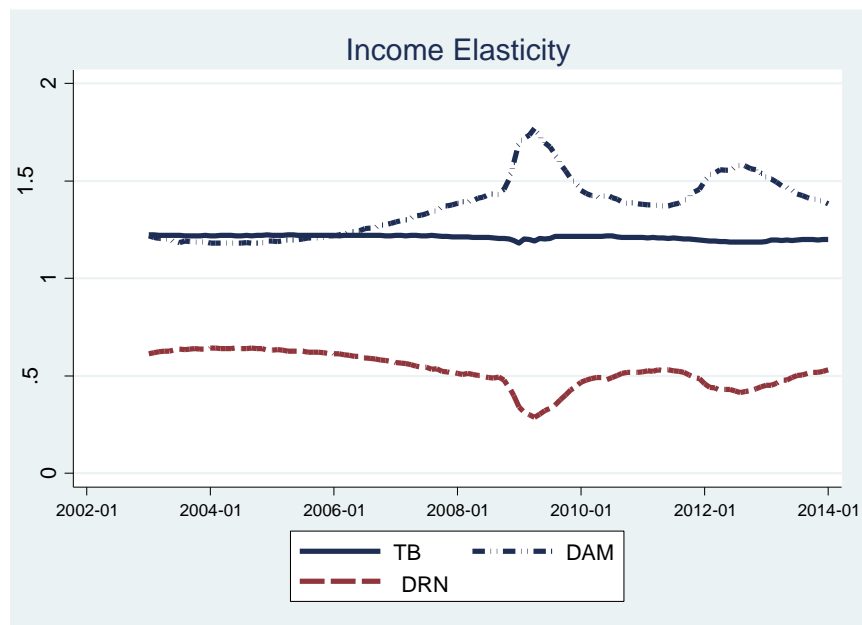


Table 2.3

Estimated price elasticities for the monetary assets

Monetary Assets i	η_{i1}	η_{i2}	η_{i3}
Transaction balances	-0.511041 (-24.0263)	-0.142338 (-3.83083)	-0.346621 (-15.5284)
Deposits with agreed maturity	-0.245840 (-3.94550)	-0.461104 (-4.20578)	-0.293057 (-5.75193)
Deposits redeemable at notice	-0.376363 (-16.4524)	-0.196048 (-6.21254)	-0.427589 (-22.7411)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Figure 2.4

Marshallian price elasticity of monetary assets with respect to the user-costs

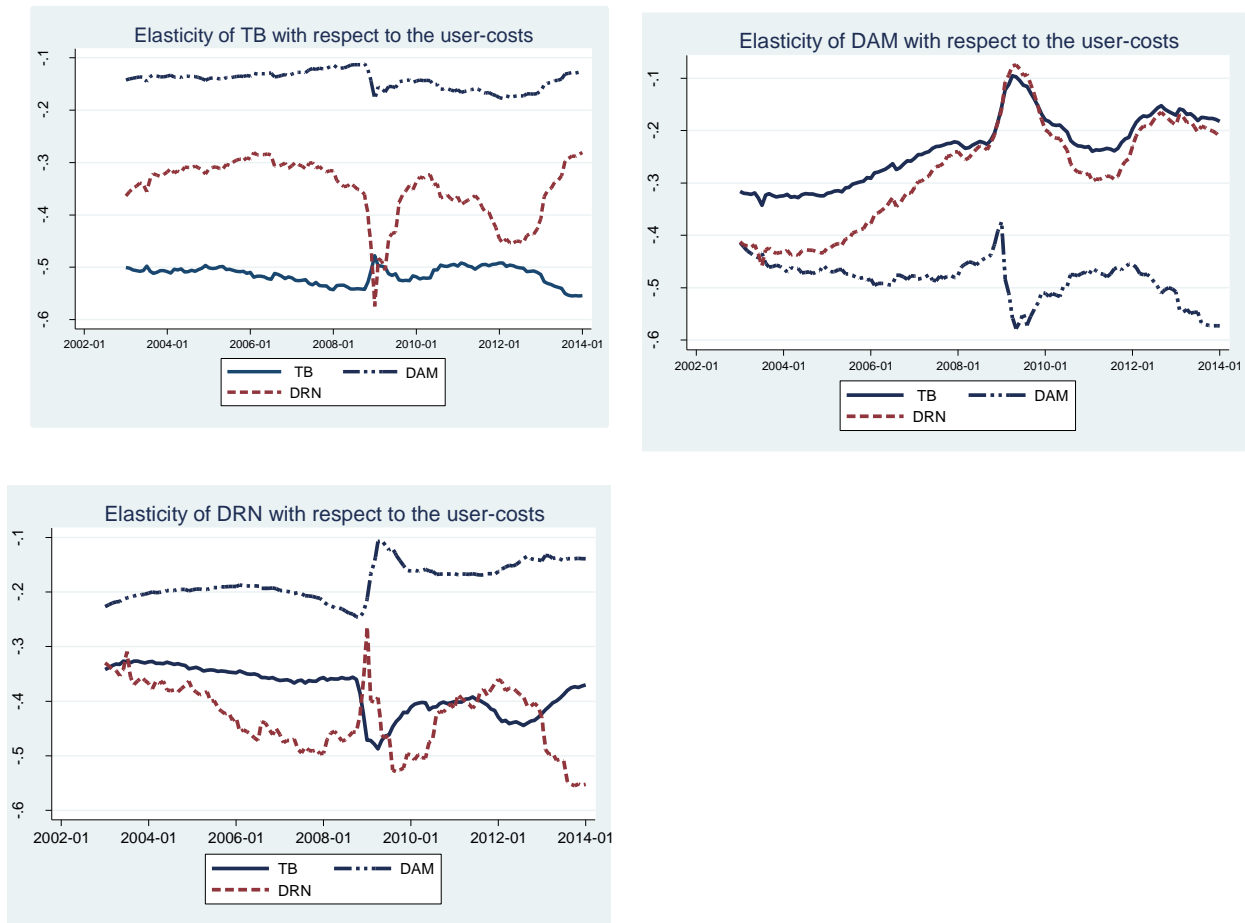


Table 2.4

Estimated Allen elasticity of substitution for the monetary assets

Monetary Assets i	σ_{i1}^a	σ_{i2}^a	σ_{i3}^a
Transaction balances	-0.063959 (-1.10478)	0.617414 (3.24950)	0.262455 (7.48267)
Deposits with agreed maturity		-0.596014 (-1.02958)	0.538153 (6.54606)
Deposits redeemable at notice			-0.640232 (-12.9860)

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Figure 2.5

Allen elasticity of substitution

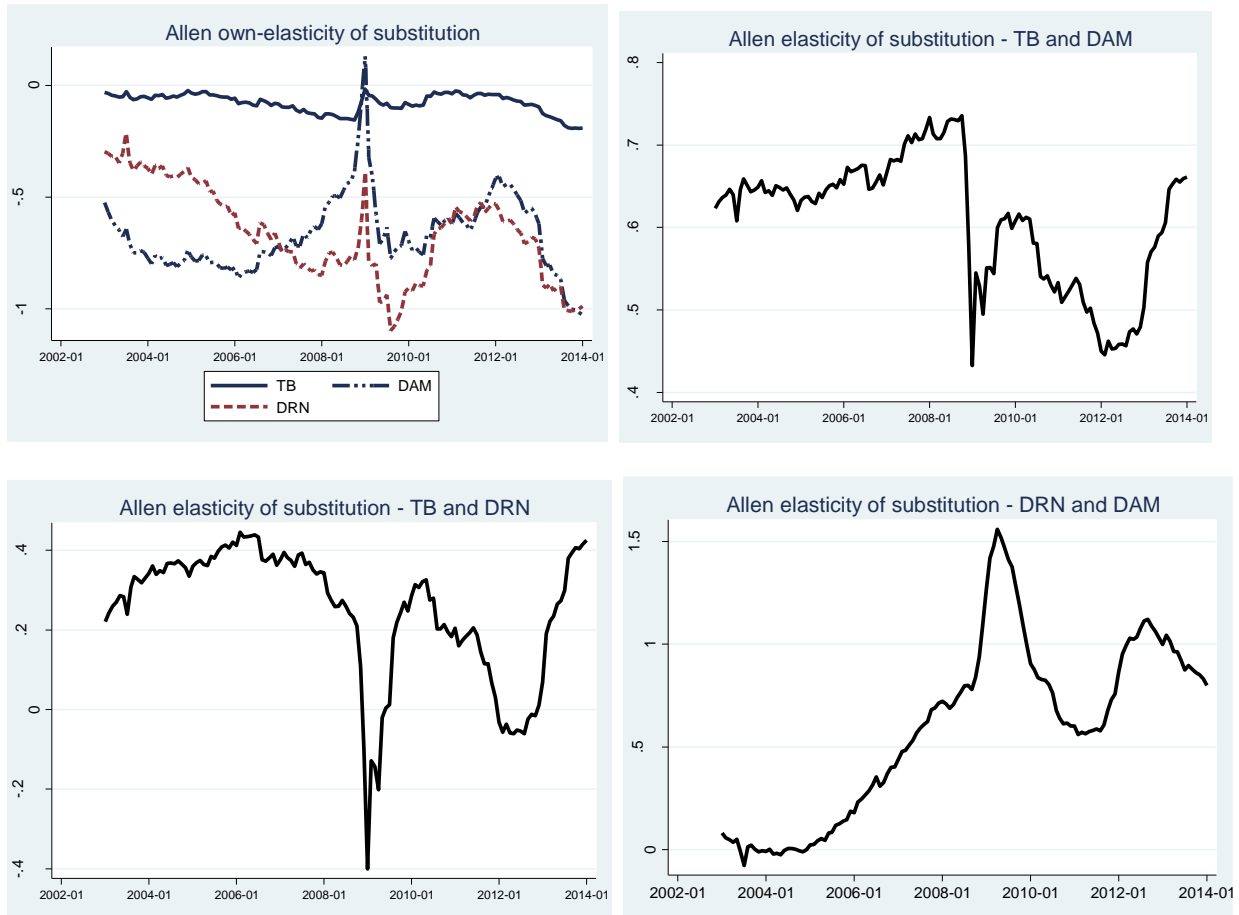


Table 2.5

Estimated Morishima elasticity of substitution for the monetary assets

Monetary Assets i	σ_{i1}^m	σ_{i2}^m	σ_{i3}^m
Transaction balances		0.271938 (3.34982)	0.130272 (5.40248)
Deposits with agreed maturity	0.288293 (1.58051)		0.269461 (1.89831)
Deposits redeemable at notice	0.327957 (13.3180)	0.428121 (9.60369)	

Numbers in parentheses are t-values. Sample period: 2003-01 to 2014-01

Figure 2.6a

Morishima Elasticity of Substitution for (i) TB and DAM when the user-cost price of DAM changes; (2) TB and DRN when the user-cost of DRN changes

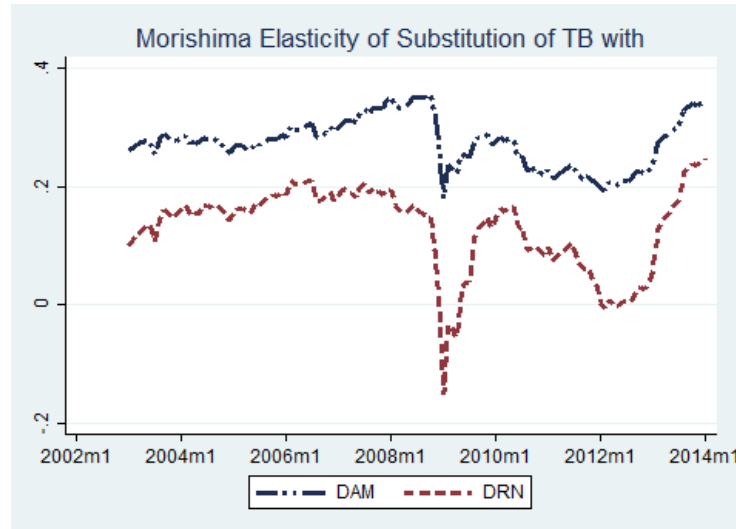


Figure 2.6b

Morishima Elasticity of Substitution for (i) DAM and TB when the user-cost price of TB changes; (2) DAM and DRN when the user-cost of DRN changes

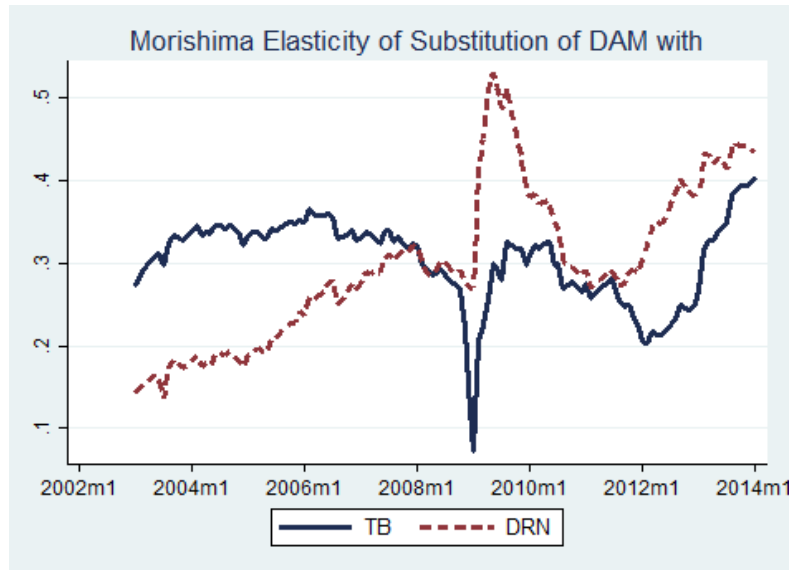
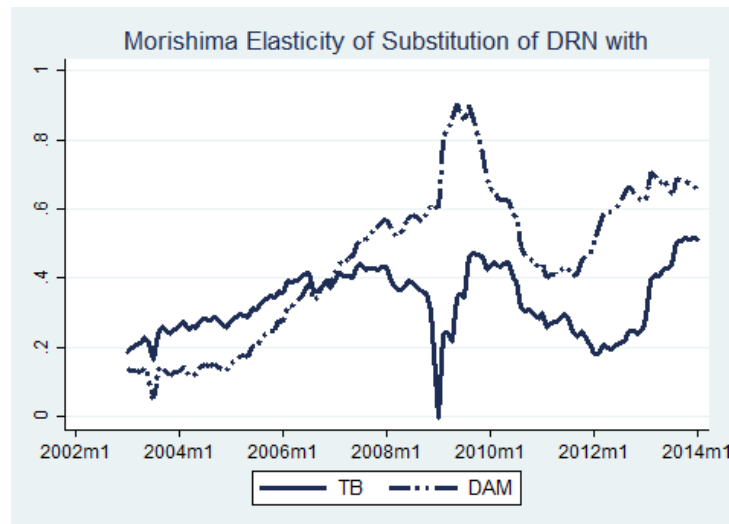


Figure 2.6c

Morishima Elasticity of Substitution for (i) DRN and TB when the user-cost price of TB changes; (2) DRN and DAM when the user-cost of DAM changes



4. Conclusion

This paper examines the monetary services in the European Monetary Union (EMU) of 11 countries. We aggregated over the monetary services of currency in circulation, overnight deposits, and deposits with agreed maturity and deposits redeemable at notice. The demand of these monetary services is analyzed to study the degree of substitutability of these monetary assets.

The monetary assets are analyzed within the framework of a representative consumer's utility function, using Barnett's (1983) locally flexible functional form, the minflex Laurent Indirect utility function. The monetary assets are analyzed for their degree of substitutability using Allen elasticity of substitution and Morishima elasticity of substitution. The results show that the monetary assets are weak substitutes, far from perfect substitutes. As a result, the theoretically correct monetary aggregate cannot be linear, and certainly cannot be simple sum.

The analysis of elasticities with respect to the asset's user-cost prices shows that: (i) transaction balances and deposits redeemable at notice are income elastic, (ii) the three monetary services are user-cost price inelastic, and (iii) Both Allen elasticity of substitution and Morishima elasticity of substitution show that the monetary services for a union of eleven countries of European Monetary Union are not perfect substitutes. This does not satisfy the necessary condition for the simple sum aggregates that components should be perfect substitutes. Hence the simple sum monetary aggregates for this union will be theoretically inconsistent and will distort measurement of the monetary aggregate.

Chapter 3

Nominal GDP and Divisia M2 Monetary aggregate: In Markov Regime Switching Approach

Introduction

The recent financial crisis has resulted in reduction in employment and incomes. This has led to a number of economists have argue the monetary policies of the central banks. Many economists, prominent of them are Crook (2011), Romer (2011) Woodford (2013) have suggested that the US Federal Reserve and the European Central Bank can adopt strategies to smooth out the fluctuations in nominal output. Bean (1983, 2013) suggested that nominal income targeting would be more productive compared to the inflation targeting. It also shows that policy of targeting nominal income produces an optimal response to demand shocks and to productivity shocks if labor supply is inelastic. Sumner (1995, 2014) argues that stable nominal GDP growth stabilizes employment and limits asset market instability. Nominal income targeting can be expected to help limit asset price bubbles, by cutting the source of funds and availability of credit created the bubbles.

Another argument relates to public debt in Euro area, the sustainability of public debt or nominal debt to nominal GDP ratio. Turner (2013) argues that targeting the nominal GDP would help predict the future debt to GDP ratio; and in turn help better coordinate the monetary and fiscal policy.

Hallet (2015) argues that the measurement of real incomes and output gaps are difficult, this is especially more challenging in case of economic union. This limitation is not only to the nominal income targeting but also to Taylor rules. The nominal income targeting is less effective if the distribution of income is more unequal (or the share of capital in national income increases). The nominal income targeting approach is seen to have both merits and drawbacks. In this analysis, the relationship between the nominal GDP and Divisia M2 monetary aggregate is studied in the case of Markov regime switching model initially proposed

by Hamilton (1989). In Hamilton (1989) the US real income was studied for the changes in the regime using Markov chain process.

Feldstein and Stock (1993) studied the possibility of using M2 monetary aggregate as a target the quarterly growth of nominal GDP, they found the relationship between M2 aggregate and nominal GDP to be strong and the use of M2 aggregate in estimating nominal GDP is seen to reduce the annual GDP variance. Belongia and Ireland (2015) have derived an approach to target the nominal GDP using P-star model, originally outlined by Working (1923). Barnett, Chauvet and Leiva-Leon (2015) have developed a dynamic factor model approach to nowcast nominal GDP growth using Divisia M3 monetary aggregate into the mode.

In this analysis, the focus is to analyze the contribution of the Divisia M2 monetary aggregate in prediction of the change in the regime of the nominal GDP for the eleven European Monetary Union countries. In this we proceed with discussing the model, the data and variables followed by results and conclusion.

Model

Let y_t denote stationary time series which is described by the first order autoregression

$$y_t = c_1 + \phi y_{t-1} + \varepsilon_t \quad (19)$$

with $\varepsilon_t \sim N(0, \sigma^2)$ which is assumed to describe the observed data for $t = 1, 2, \dots, t_0$ Further assumed that

at time t_0 there was a significant change in the average level of the series, the data now is described according to

$$y_t = c_2 + \phi y_{t-1} + \varepsilon_t \quad (20)$$

for $t = t_{0+1}, t_{0+2}, \dots$. Let S_t is a random variable that is a result of an institutional change, that is, unobserved state of the system.

$$y_t = c_{st} + \phi y_{t-1} + \varepsilon_t \quad (21)$$

$$s_t = 1 \quad t = 1, 2, \dots, t_0$$

$$s_t = 2 \quad t = t_{0+1}, t_{0+2}, \dots$$

The probabilistic model that would describe the behavior of change from $s_t = 1$ to $s_t = 2$, can be specified by tow state Markov chain

$$\Pr(s_t = j / s_{t-1} = i, s_{t-2} = k, \dots, y_{t-1}, y_{t-1}, \dots) = \Pr(s_t = j / s_{t-1} = i) = p_{ij} \quad (22)$$

Assuming that S_t is not directly observable, but can only be inferred from y_t . The parameters necessary to fully describe the probability law governing y_t are, the variance of Gaussian innovation σ^2 , the autoregressive coefficient ϕ , the two intercepts c_1 and c_2 and two state transition probabilities p_{11} and p_{22} . Lindgren (1978) and Baum et al (1980) estimated the model (3) and (4) with no autoregressive element ($\phi = 0$).

The conditional regime probability distribution given all observations up to time t

$$\xi_{jt} = \Pr(s_t = j / \Omega_t; \theta) \quad (23)$$

for $j = 1, 2$ The inference about the value of S_t based on observations of y_t , this inference take the form of probabilities ξ_{jt} , $\Omega_t = \{y_t, y_{t-1}, \dots, y_1, y_0\}$ are observations of y_t and θ is a vector of population

parameters $\boldsymbol{\theta} = (\sigma, \phi, c_1, c_2, p_{11}, p_{22})'$. The inference is performed iteratively for $t = 1, 2, \dots, T$ with step t accepting as input values

$$\xi_{it-1} = \Pr(s_{t-1} = i / \Omega_{t-1}; \boldsymbol{\theta}) \quad (24)$$

ξ_{it} can be expressed as

$$\xi_{it} = \frac{\sum_{j=1}^2 p_{ij} \xi_{it-1} \eta_{jt}}{f(y_t / \Omega_{t-1}; \boldsymbol{\theta})} \quad (25)$$

To perform this iteration η_{jt} and $f(y_t / \Omega_{t-1}; \boldsymbol{\theta})$ is required. The densities under two regimes is given by

$$\eta_{jt} = f(y_t / s_t = j, \Omega_{t-1}; \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[\frac{-(y_t - c_j - \phi y_{t-1})^2}{2\sigma^2} \right] \quad (26)$$

And the conditional density of the t^{th} observation is given by

$$f(y_t / \Omega_{t-1}; \boldsymbol{\theta}) = \sum_{i=1}^2 \sum_{j=1}^2 p_{ij} \xi_{i,t-1} \eta_{jt} \quad (27)$$

For the specified values of $\boldsymbol{\theta}$, the iteration ξ_{it} would allow to evaluate the conditional log likelihood of the observed data

$$\log f(y_1, y_2, \dots, y_T / y_0; \boldsymbol{\theta}) = \sum_{t=1}^T \log f(y_t / \Omega_{t-1}; \boldsymbol{\theta}) \quad (28)$$

An estimate of the values of $\boldsymbol{\theta}$ can then be obtained by maximizing (28). In the estimation we have used one, two, three and four autoregression process and Divisia M3 monetary aggregate in the estimation of the nominal GDP for eleven European Monetary Union countries.

Data and Variables:

This paper has used quarterly data from first quarter 2003 to first quarter 2014. Our data for the monetary services, the populations and consumer prices of the eleven EMU countries are acquired from the Statistical Data Warehouse, which is the source provided on the ECB's website. Apart from the ECB, the central banks of the member countries are also sources of some of our data. The household data on deposits and interest rates are from the ECB for deposits at Monetary and Financial Institutions (MFI). The currency data are from the central banks of member countries. The Divisia M2 aggregate index is constructed for the 11 European Monetary Union (EMU) countries which include: Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia. The Divisia M2 aggregate for these eleven EMU countries will be referred to as EMU-11 in the paper. The construction of the Divisia index for the EMU-11 is discussed in detail in Chapter 1. The quarterly nominal GDP data is acquired from the Database of the Eurostat. The nominal GDP and the Divisia monetary aggregate for EMU-11 is logarithmically differenced to attain stationarity. The figure 3.1 shows the nominal gdp for the EMU-11 countries and figure 3.2 shows the year-over-year percentage change of nominal GDP for the EMU-11 countries.

Figure 3.1: Nominal GDP for the union of EMU-11 countries.

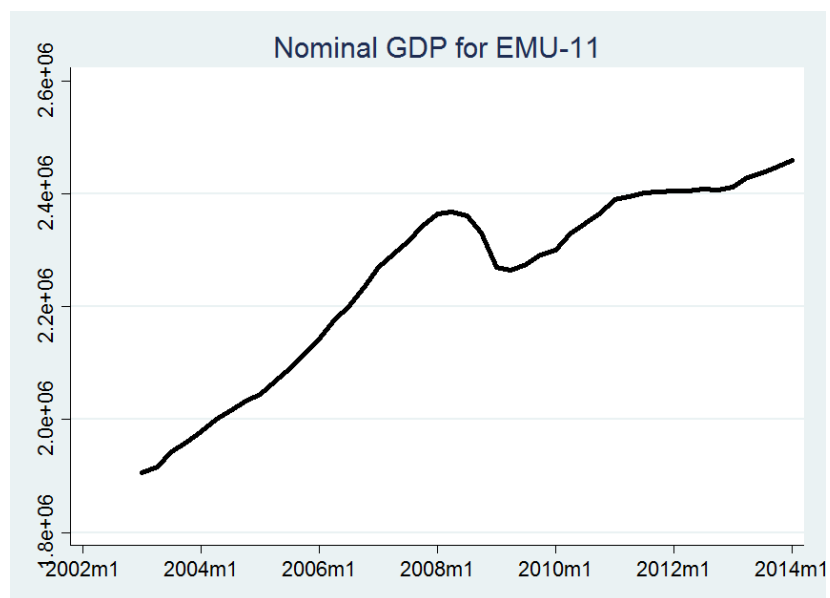
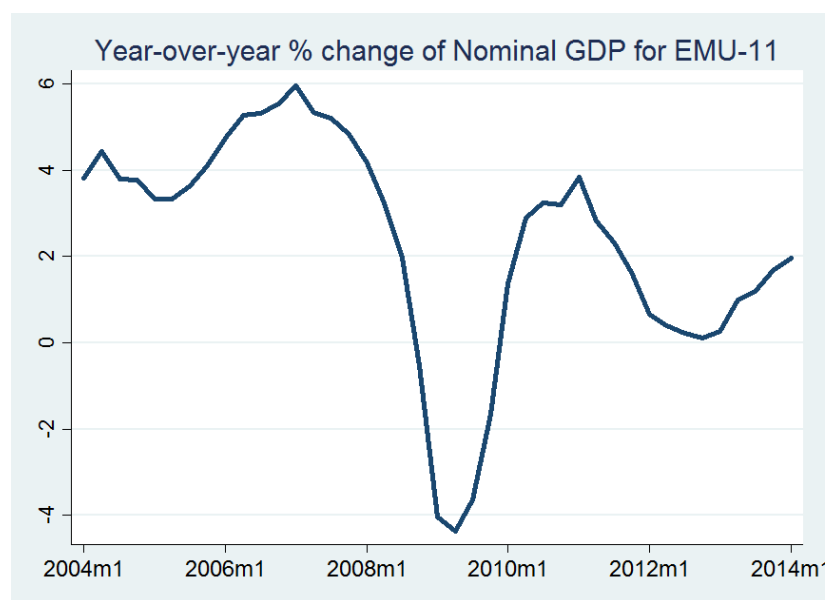


Figure 3.2: Year-over-year percentage of nominal GDP for the union of EMU-11 countries.



Results:

The Maximum likelihood estimates of the model are in Table 3.1 and 3.2. Table 3.1 represents the present the maximum likelihood estimates of the results of a model without Divisia M2 monetary aggregate. The parameter estimate of the autoregressive variable ϕ_3 is small indicating that it does not explain the GDP to a large extent. The probabilistic parameter estimate p_{22} has a large standard error. Table 3.2 shows the maximum likelihood estimation results of the model with Divisia M2 monetary aggregate. In this model the parameter for the 10% change in the monetary aggregate explains significantly 2.2% change in nominal GDP.

The table 3.3 and 3.4 present the results for the transition probabilities for the model without and with Divisia M2 monetary aggregate. The model without the monetary aggregate show a very low probability of persistence of the state 2, that is, 3.46E-10. Whereas the model with the monetary aggregate shows a higher probability of persistence of the state 2, that is, 0.46. The estimated regime probabilities with two models is shown in figure 3.3 and 3.4. The figure 3.4 shows a longer persistence in state 2 compared to figure 3.3.

Table 3.1: Parameter estimates for model without the Divisia M2 monetary aggregate

Parameter	Estimate	SE	t-values	p-values
c_1	0.257250	0.093959	2.737897	0.0062
c_2	-0.617434	0.191849	-3.218338	0.0013
p_{11}	3.671071	1.151669	3.187609	0.0014
p_{22}	21.78538	54180.14	0.000402	0.9997
σ	-1.813087	0.127770	-14.19028	0.0000
ϕ_1	0.720645	0.198951	3.622228	0.0003
ϕ_2	0.210658	0.245904	0.856669	0.3916
ϕ_3	-0.071936	0.210395	-0.341910	0.7324
ϕ_4	-0.173536	0.182219	-0.952353	0.3409

Table 3.2: Parameter estimates for model that used the Divisia M2 monetary aggregate

Parameter	Estimate	SE	t-values	p-values
c_1	0.264407	0.136965	1.930464	0.0535
c_2	-0.607012	0.202218	-3.001763	0.0027
p_{11}	3.675581	1.169619	3.142547	0.0017
p_{22}	0.125111	1.541155	0.081180	0.9353
$dm2$	0.224539	0.096323	2.331103	0.0197
σ	-1.882354	0.129874	-14.49366	0.0000
ϕ_1	0.437401	0.191806	2.280427	0.0226
ϕ_2	0.176637	0.187085	0.944152	0.3451
ϕ_3	0.365132	0.193249	1.889442	0.0588
ϕ_4	-0.184735	0.185125	-0.997893	0.3183

Table 3.3: Constant transition probabilities for the estimation without the Divisia M2 monetary aggregate,

$$P(i, k) = P(s_t = k / s_{t-1} = i) \text{ row} = i, \text{ column} = j$$

	1	2
1	0.975182	0.024818
2	1.000000	3.46E-10

Table 3.4: Constant transition probabilities for the estimation with the Divisia M2 monetary aggregate,

$$P(i, k) = P(s_t = k / s_{t-1} = i) \text{ row} = i, \text{ column} = j$$

	1	2
1	0.975291	0.024709
2	0.531237	0.468763

Figure 3.3: Markov switching regime probabilities for nominal GDP for model without the Divisia M2 monetary aggregate

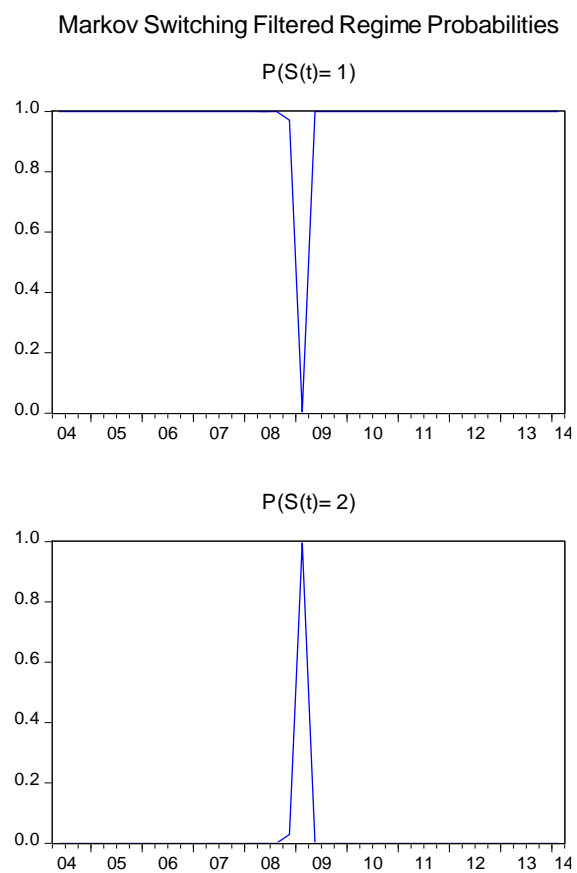
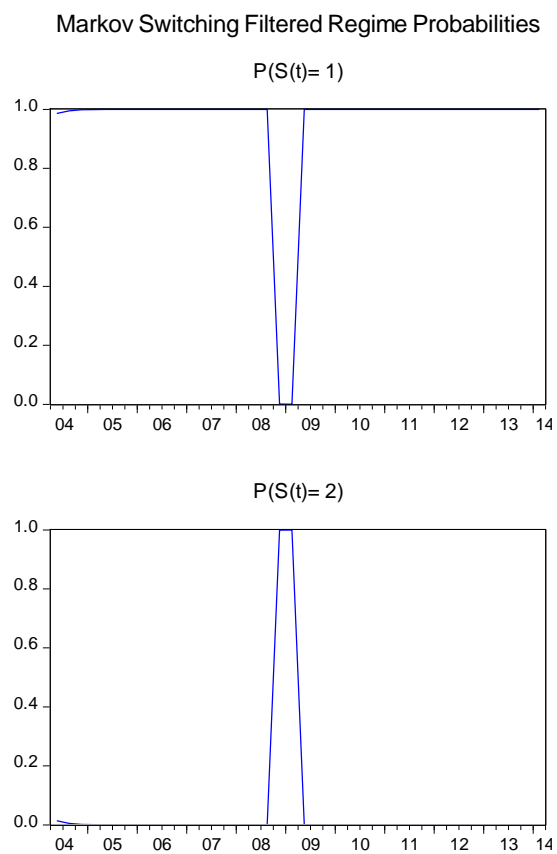


Figure 3.4: Markov switching regime probabilities for nominal GDP for model with the Divisia M2 monetary aggregate



Conclusion:

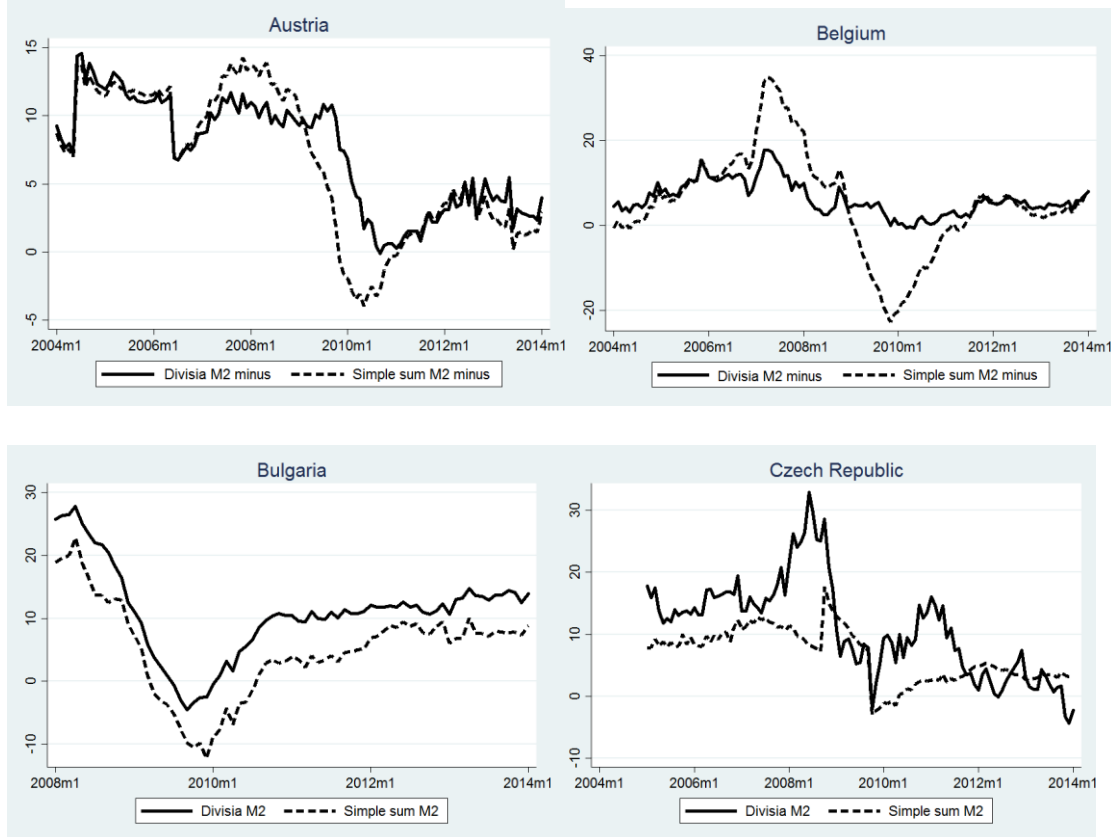
Euro area is an economic union where it offers the countries of Euro zone the possibility of targeting their nominal incomes and also allowing the national policy makers to target their own national incomes. The member countries of Euro zone are sovereign and they control nearly all the policy instruments outside monetary policy. Binder and Gross (2013) have estimated regime switching models for real income for a few Euro area countries.

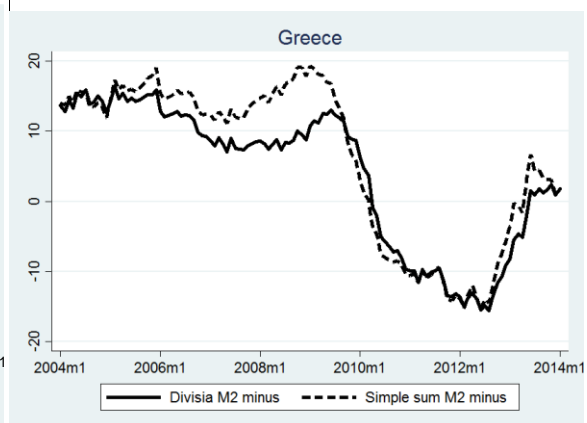
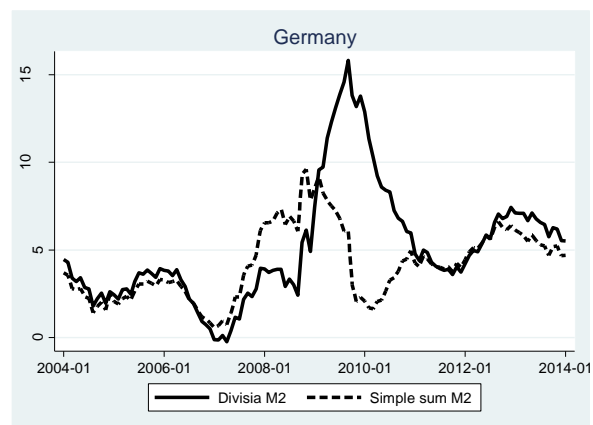
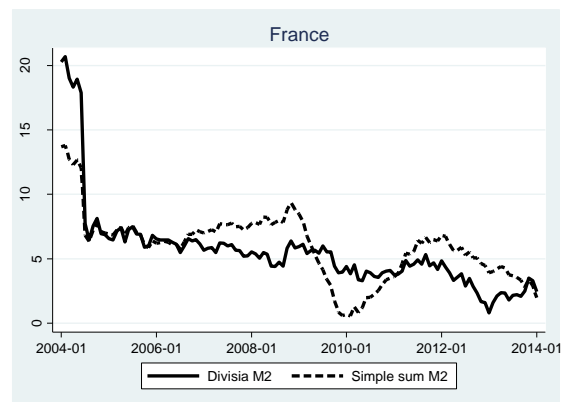
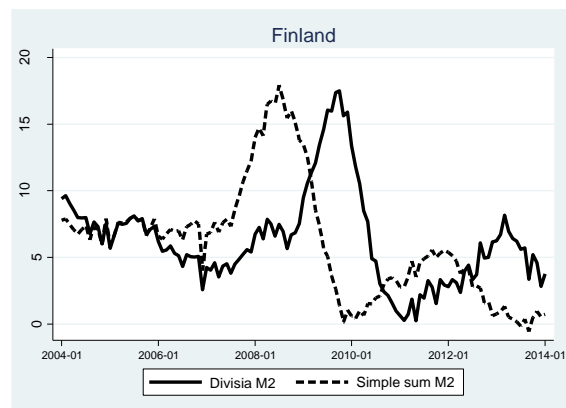
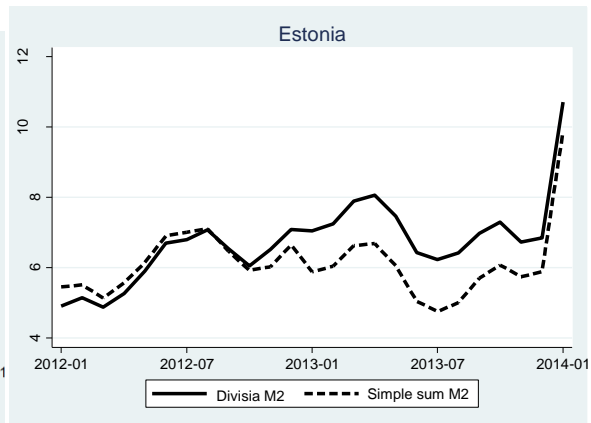
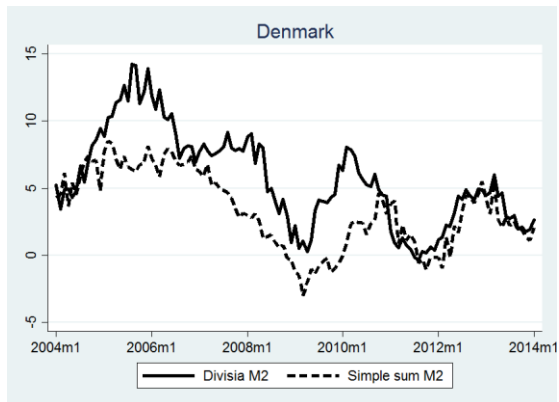
In this analysis for the union of eleven European Monetary Union countries, the inclusion of Divisia M2 monetary aggregate into the model along with the nominal GDP autoregressive variables shows an increase in performance of the model, this is indicated by an increase in the probability of persistence of nominal GDP in state 2 (up to 2 quarters). This result is indicative and may need further substantiation. I intend to perform the similar analysis for the eleven individual countries of the European Monetary Union and also extend the period of analysis.

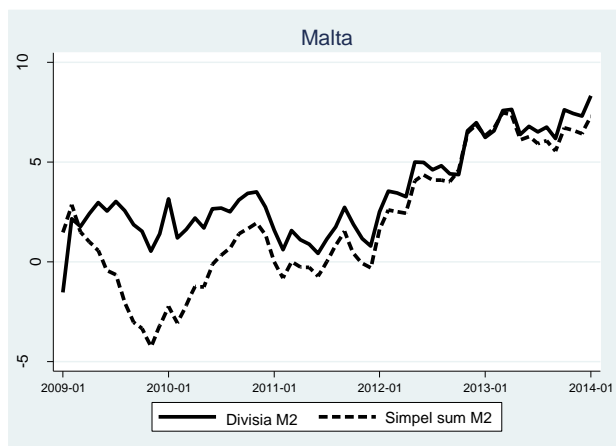
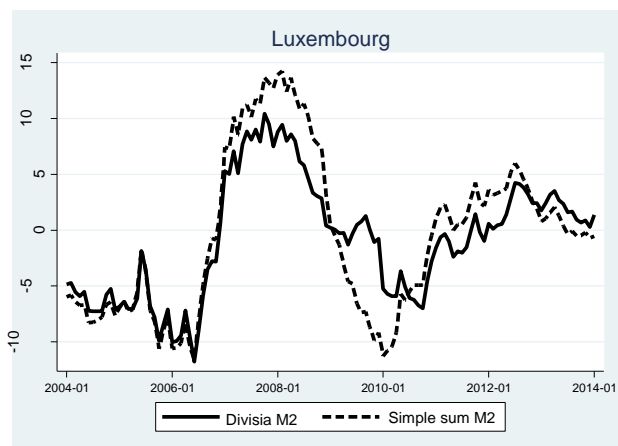
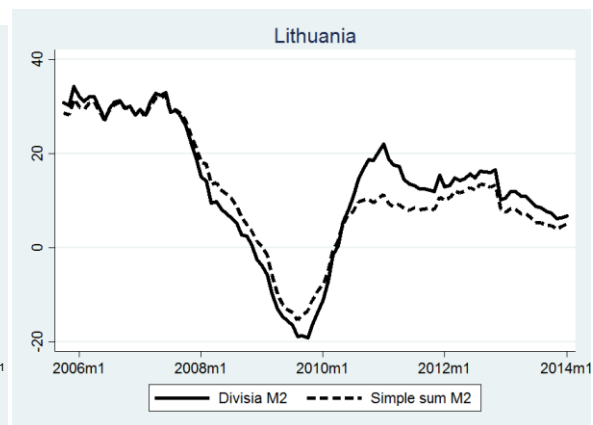
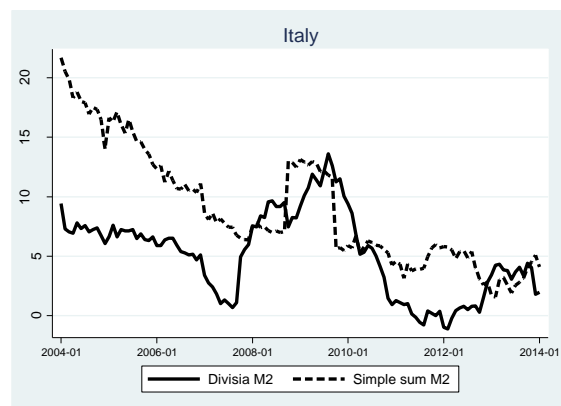
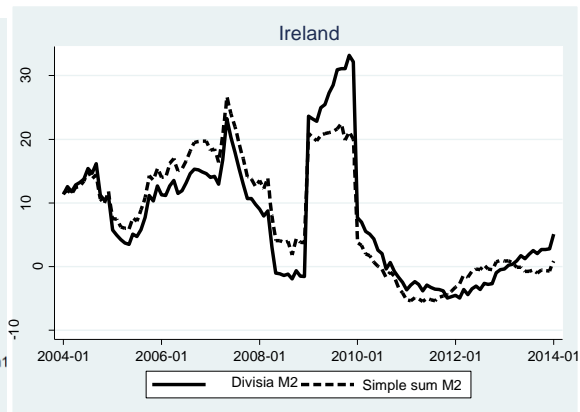
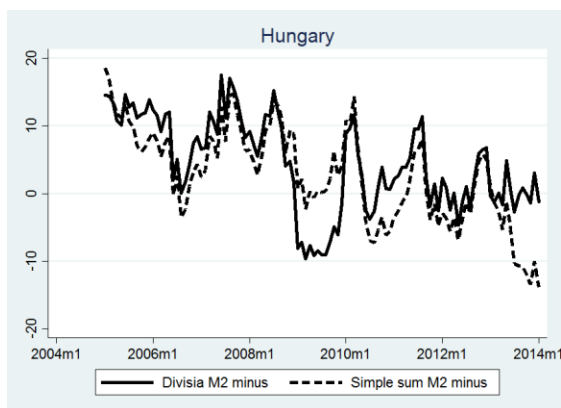
Appendices

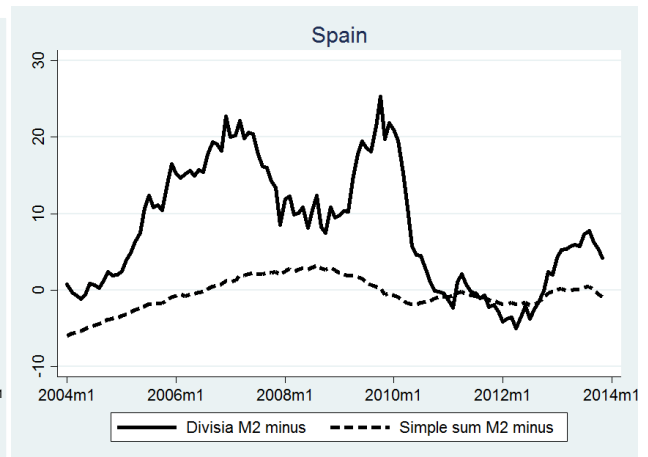
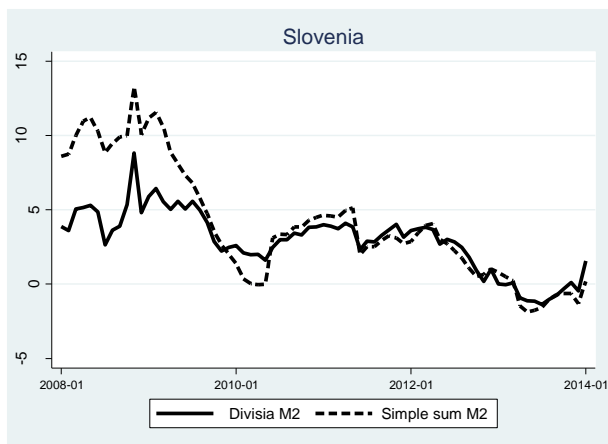
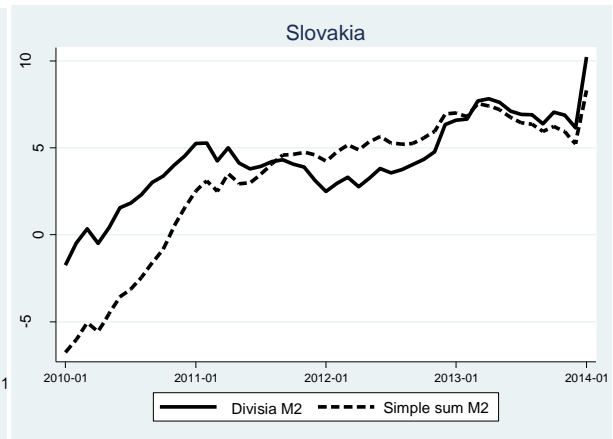
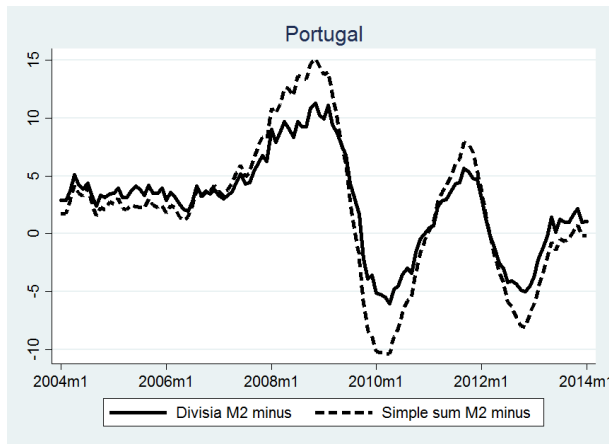
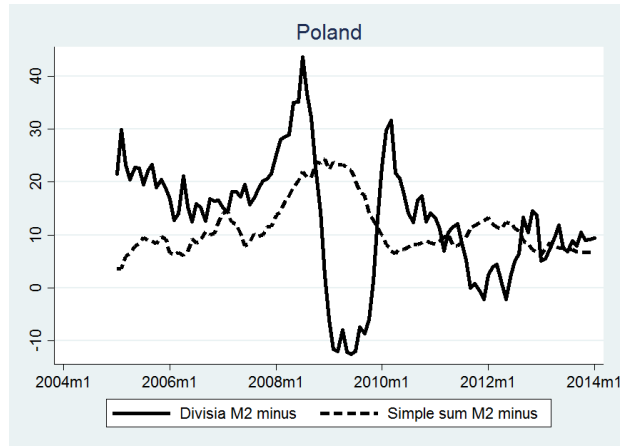
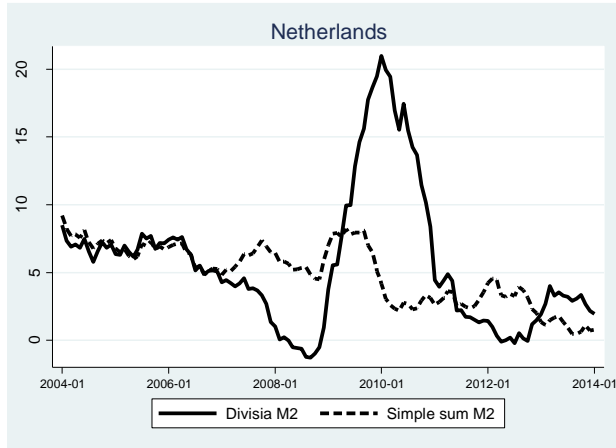
Appendix A1

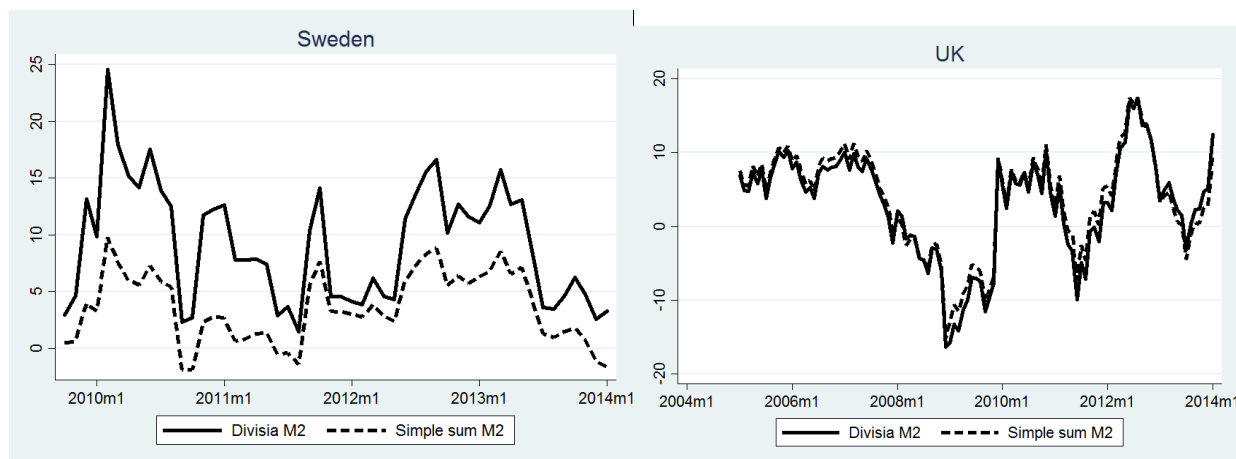
Year-over-year percentage change of the Divisia and simple sum monetary aggregates for Euro Area countries.











Appendix A2

Definitions:

Monetary and financial institutions (MFI) from the ECB Glossary: MFIs are Central Bank, resident credit institutions as defined by community law, and other resident financial institutions whose business is to receive deposits and /or close substitutes for deposits from entities other than MFIs and for their own account to grant credits and/or make investments in securities.

Overnight deposits from the ECB Glossary, deposits with next-day maturity: This instrument category comprises mainly those sight/demand deposits that are fully transferable by check or similar instrument. It also includes non-transferable deposits that are convertible on demand or by close of business the following day. Overnight deposits are included in M1 and hence in M2 and M3.

Deposits redeemable at notice (DRN) from the ECB Glossary: These deposits are savings deposits for which the holder must respect a fixed period of notice before withdrawing the funds. In some cases there is the possibility of withdrawing on demand a certain fixed amount in a specified period or of early withdrawal subject to the payment of a penalty. Deposits redeemable at a period of notice up to three months

are included in M2 and hence in M3, while those with a longer period of notice are part of the non-monetary longer term financial liabilities of the MFI sector.

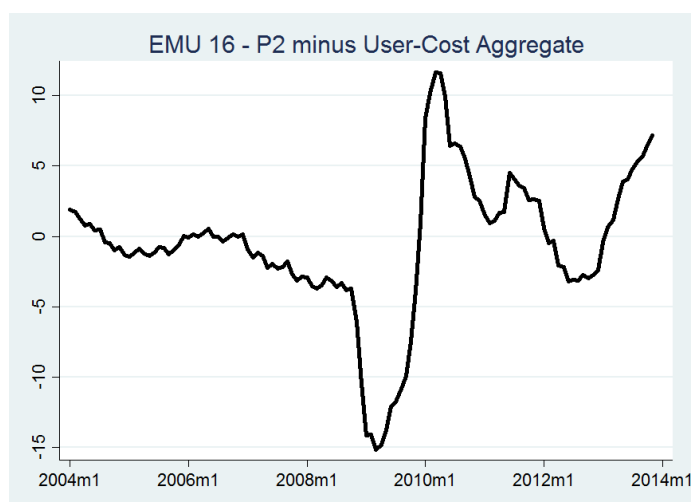
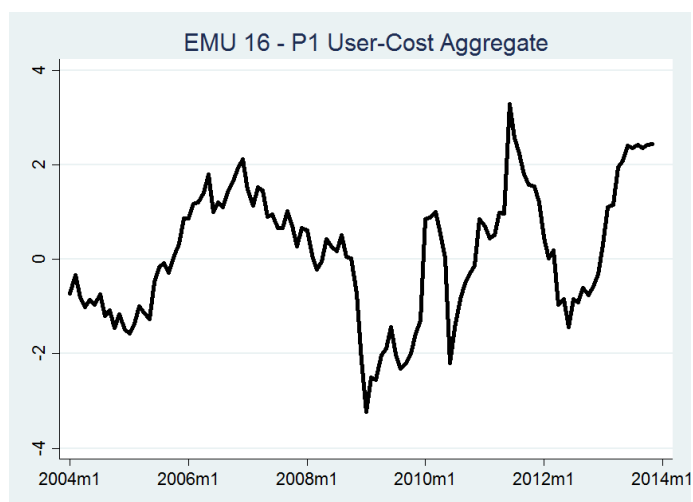
Deposits with an agreed maturity (DAM) from the ECB Glossary: These deposits are mainly time deposits with a given maturity that, depending on national practices, may be subject to the payment of a penalty in the event of early withdrawal. Some non-marketable debt instruments, such as non-transferable retail certificates of deposit, are also included. Deposits with an agreed maturity of up to two years are included in M2 and hence in M3, while those with an agreed maturity of over two years are included in the non-monetary long term financial liabilities of the MFI sector.

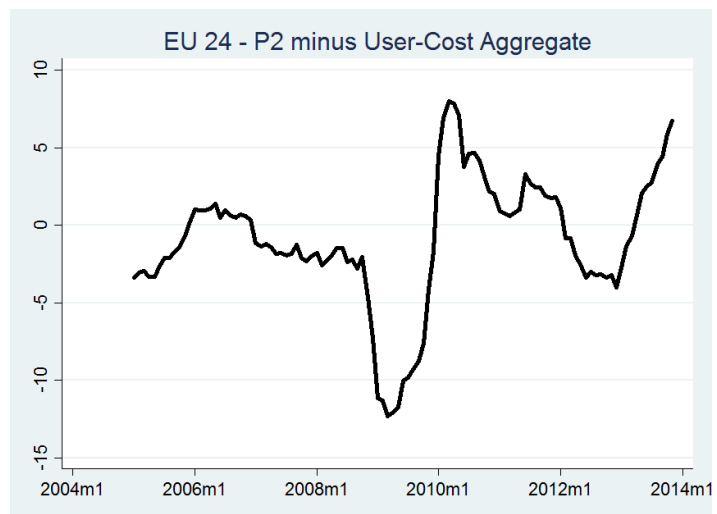
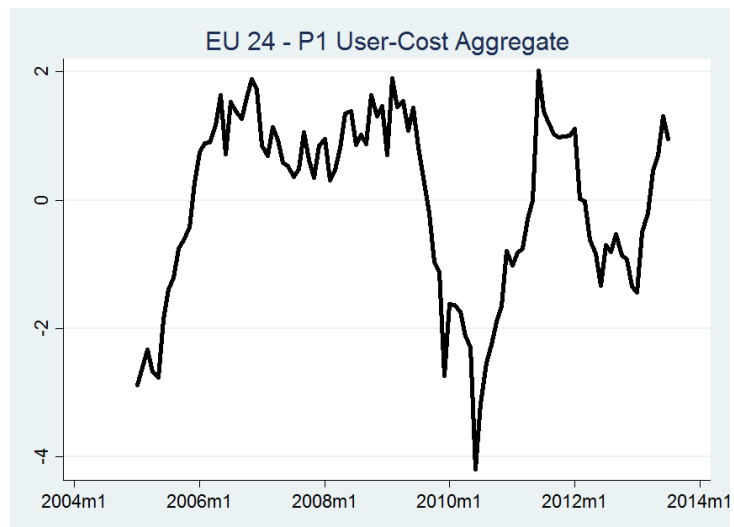
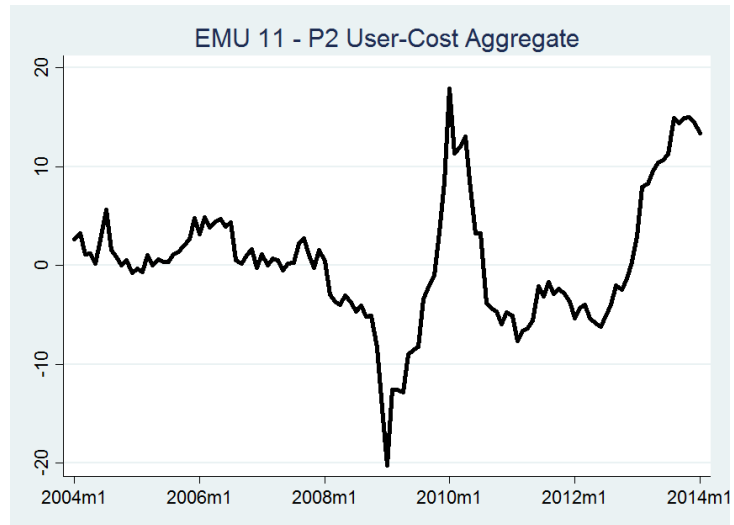
Non-profit institutions serving households (NPISH) from the Eurostat Glossary: These institutions make up an institutional sector in the context of national accounts consisting of non-profit institutions which are not mainly financed and controlled by government and which provide goods or services to households for free or at prices that are not economically significant. Examples include churches and religious societies, sports and other clubs, trade unions, and political parties. NPISH are private, non-market producers which are separate legal entities. Their main resources, apart from those derived from occasional sales, are derived from voluntary contributions in cash or in kind from households in their capacity as consumers, from payments made by general governments, and from property income.

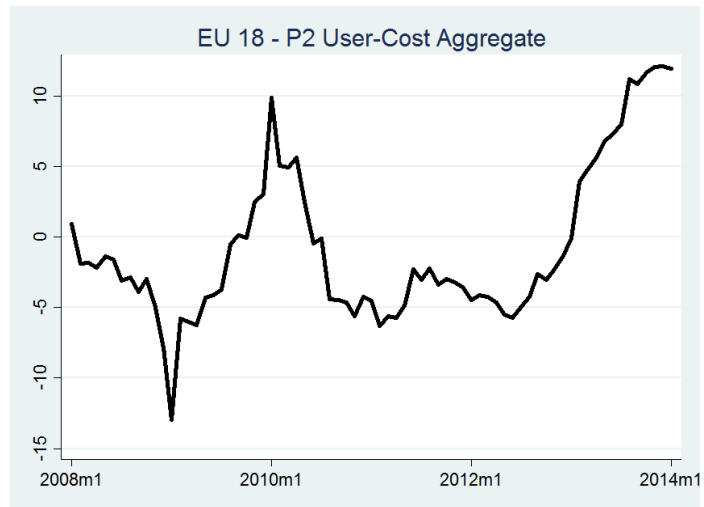
Non-financial corporation (NFC) from the ECB Glossary: These firms are corporation or quasi-corporation that is not engaged in financial intermediation but is active primarily in the production of market goods and non-financial services.

Appendix A3

The year-over-year percentage change for user-cost aggregate for EMU and EU unions.





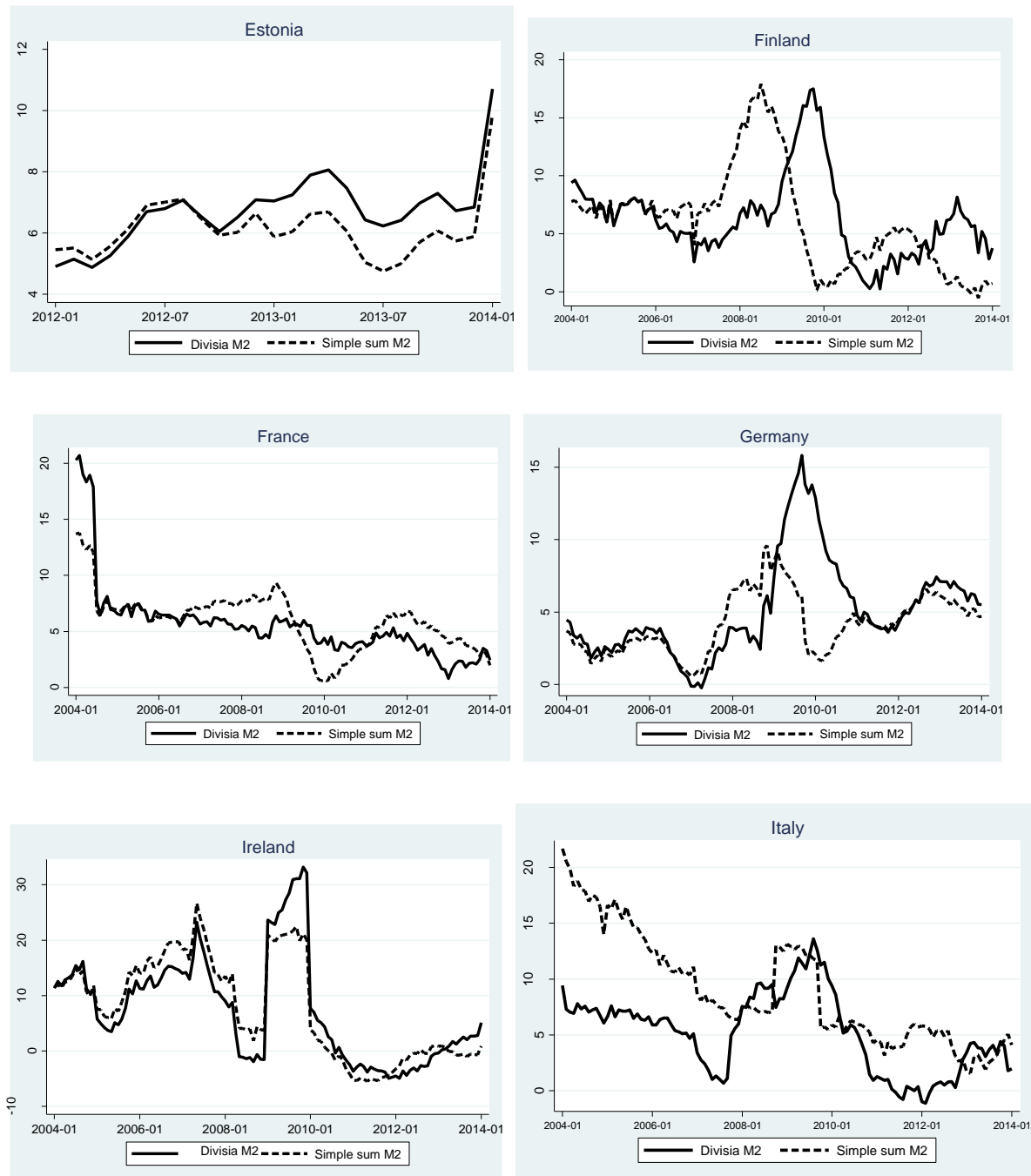


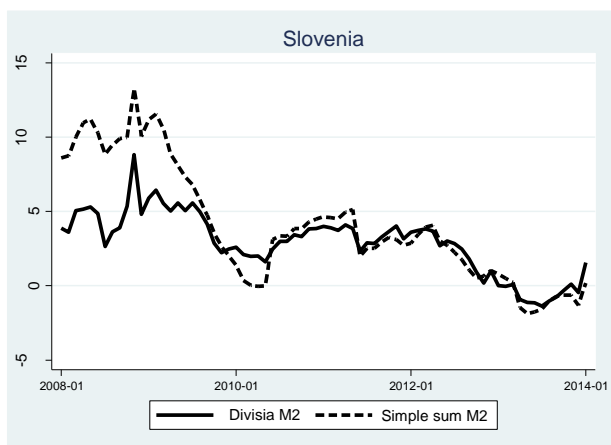
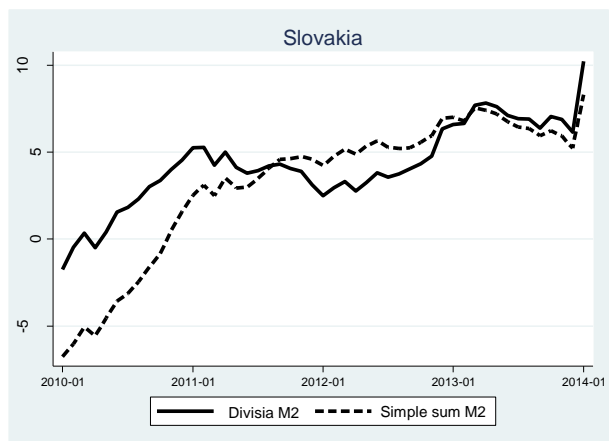
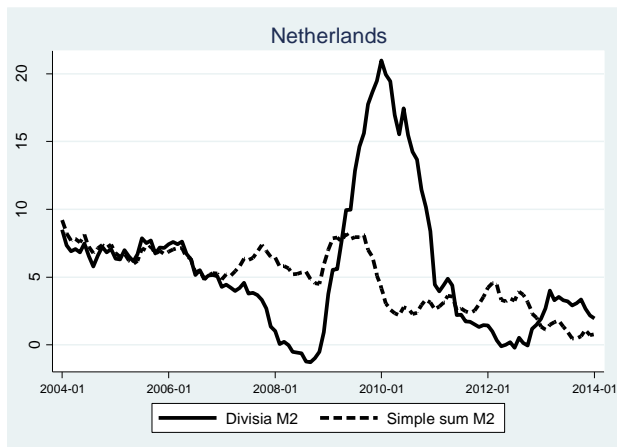
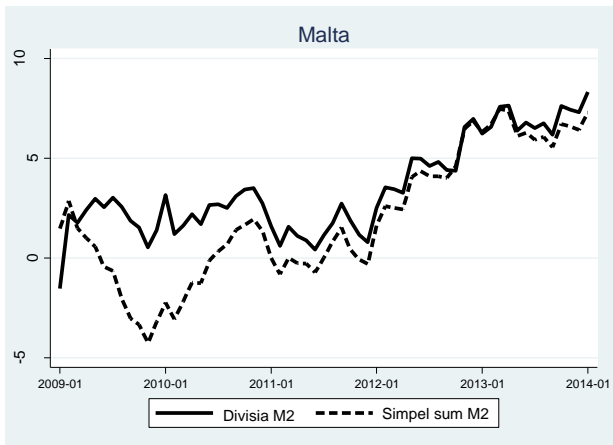
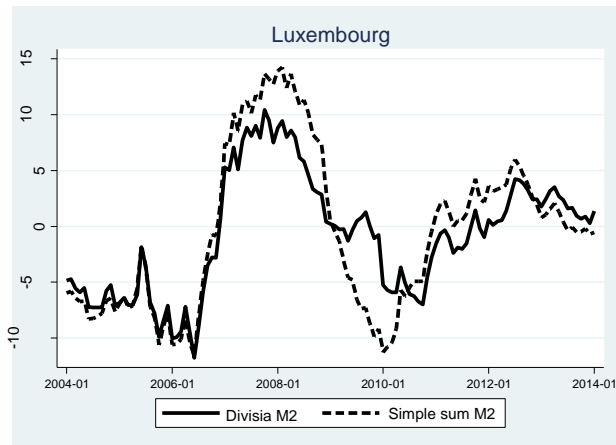
Appendix B

Appendix B1

Figure 2.1

Year-over-year percentage change of the Divisia and simple sum monetary aggregates for EMU-11 countries.

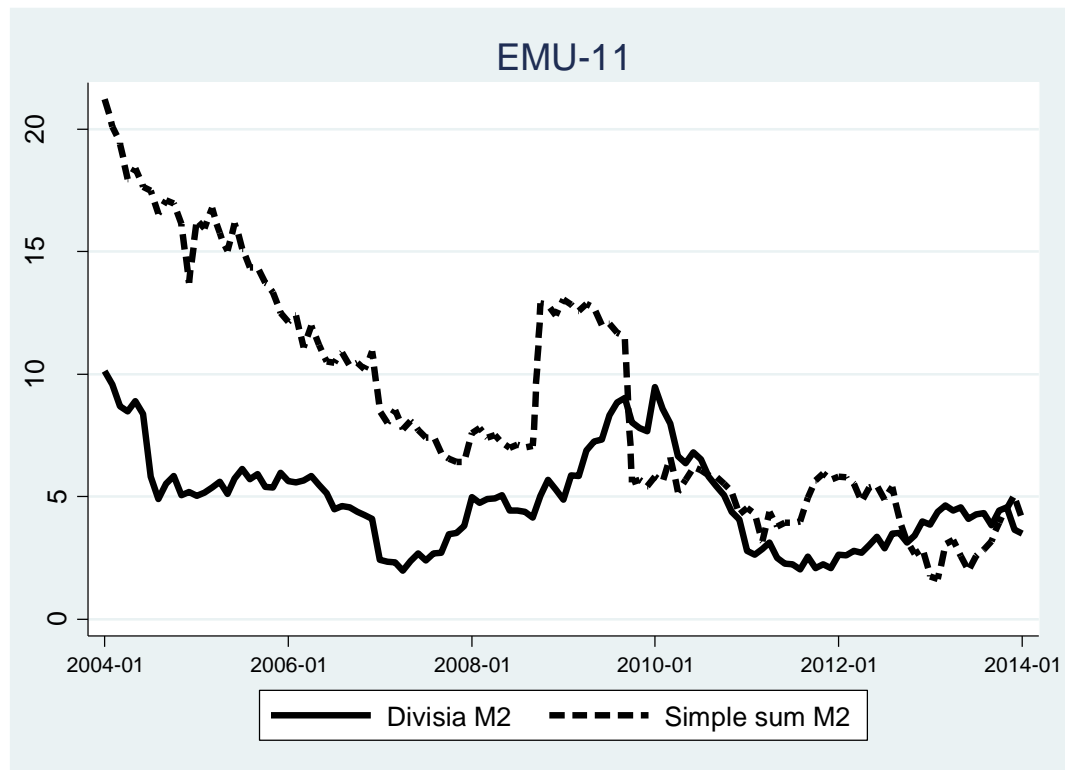




Appendix B2

Figure 2.2

Year-over-year percentage change in Divisia and simple sum monetary aggregate for the union EMU-11



References

- Barnett, W.A., 1978. The user cost of money. *Economics Letters* 1, 145--149 In: Barnett, W.A., Serletis, A., (Eds.), *The Theory of Monetary Aggregation*. (Reprinted) Elsevier, Amsterdam, pp. 6--10.
- Barnett, W.A., 1980. Economic monetary aggregates: an application of aggregation and index number theory. *Journal of Econometrics* 14, 11--48 In: Barnett, W.A., Serletis, A. (Eds.), *The Theory of Monetary Aggregation*, (Reprinted) Elsevier, Amsterdam, PP.11-48.
- Barnett, W. A ., Edward K. Offenbacher, and Paul A. Spindt (1981), "New Concepts of Aggregated Money." *Journal of Finance* 36, 497-505.
- Barnett, W.A. (1983). "New indices of money supply and the flexible Laurent demand system". *Journal of Business and Economic Statistics* 1: 7--23.
- Barnett, W A., Edward K. Offenbacher, and Paul A. Spindt. (1984) "The New Divisia Monetary Aggregates." *Journal of Political Economy* 92, 1049-85
- Barnett WA, Lee YW. 1985. The global properties of the minflex Laurent, generalized Leontief, and translog flexible functional forms. *Econometrica* 53: 1421--1437.
- Barnett, W.A., 2003. Aggregation-theoretic monetary aggregation over the euro area, when countries are heterogeneous. *European Central Bank Working Paper* no. 260. Frankfurt.
- Barnett A William, 2007. Multilateral Aggregation-Theoretic Monetary Aggregation over Heterogeneous Countries. *Journal of Econometrics*, vol 136, no 2, pp 457-482.
- Barnett, W. A. and Chauvet, M., 2011. "How better monetary statistics could have signaled the financial crisis." *Journal of Econometrics*, 161(1):6-23
- Barnett W A, 2012. *Getting it wrong. How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*. MIT Press

- Barnett, W. A., Liu J, Ryan S. Mattson R. S, and van den Noort J., 2013, 'The New CFS Divisia Monetary Aggregates: Design, Construction, and Data Sources', *Open Economies Review* 24, 101-124
- Barnett. W and D. Leiva-Leon (2016) , "Real-Time Nowcasting of Nominal GDP with Structural Breaks" *Journal of Econometrics*. Vol. 191, April, 312-324.
- Bean, Charles (1983) "Targeting Nominal Income: An Appraisal", The *Economic Journal*, Vol 93, No. 372, 806-819.
- Bean, Charles (2013) "Nominal Income Targets: an Old Wine in a New Bottle", speech given at the Institute of Economic Affairs, 27 February 2013, available at www.bankofengland.co.uk/publications/speeches/2013/speech640.pdf.
- Belongia, Michael T. and Peter N. Ireland (2015). "A 'Working' Solution to the Question of Nominal GDP Targeting," *Macroeconomic Dynamics*, April.
- Belongia, Michael T. and Peter N. Ireland (2014). "The Barnett Critique After Three Decades: A New Keynesian Analysis," *Journal of Econometrics*, November.
- Binder. M and M. Gross (2013), "Regime Switching Global Vector Auto regressive models" ECB working paper No. 1569 August 2013.
- Feldstein, Martin and James H. Stock (1993). "The Use of a Monetary Aggregate to Target Nominal GDP," NBER Working Paper No. 4304, March.
- Barten A. P. 1969. Maximum likelihood estimation of a complete system of demand equations. *European Economic Review* 1: 7-73.
- Belongia, M.T. and P.N. Ireland (2006). "The Own-Price of Money and the Channels of Monetary Transmission." *Journal of Money, Credit, and Banking* **38**: 429-45

- Belongia, M.T. and P.N. Ireland (2015a). "Interest Rates and Money in the Measurement of Monetary Policy." *Journal of Business and Economic Statistics* **332**: 255-269.
- Belongia, M.T. and P.N. Ireland (2015b). "A 'Working' Solution to the Question of Nominal GDP Targeting." *Macroeconomic Dynamics* **19**: 508-534.
- Belongia, M.T. and P.N. Ireland (2016). "Money and Output: Friedman and Schwartz Revisited." *Journal of Money, Credit and Banking* **48(6)**: 1223-1266.
- Benati, Luca (2008). "Investigating inflation persistence across monetary regimes", *Quarterly Journal of Economics*, 123(3), 1005 - 1060.
- Benati, Luca (2009), "Long run evidence on money growth and inflation", ECB Working Paper No. 1027, European Central Bank.
- Beyer, A. (2008) Euro Area Money Demand is Stable!, Mimeo presented at NCB Expert Workshop, Frankfurt am Main, 14 November.
- Blackorby, C. and R.R. Russell 1989. "Will the real elasticity of substitution please stand up?" *American Economic Review* 79, 882--888
- Boone, L. Mikol, F. and Van den Noord, P. (2004). Wealth Effects on Money Demand in EMU: Econometric Evidence, OECD Economics Department Working Papers 411.
- Bruggeman, A., G. Camba-Mendez, B. Fischer and J. Sousa (2005), "Structural filters for monetary analysis: the inflationary movements of money in the euro area", ECB Working Paper no. 470.
- Chan, W and Nautz, D. (2015) "The information content of monetary statistics for the Great Recession: Evidence from Germany" SFB 649 Discussion Paper 2015-027.
- Cherchye, L., T. Demuynck, B. D. Rock, and P. Hjerstrand (2015), "Revealed Preference Tests for Weak Separability: An Integer Programming Approach," *Journal of Econometrics* **186(1)**: 129-141.

Darvas, Z. (2015). "Does money matter in the euro area? Evidence from a new Divisia index" *Economic Letters* 133, 123-126.

Fisher, I. (1922). *The Making of Index Numbers: A Study of Their Varieties, Tests, and Reliability*, Boston: Houghton Mifflin.

Gerlach, S. and K. Assenmacher-Wesche (2006), "Interpreting euro area inflation at high and low frequencies", BIS Working Paper no. 195.

Gorman WM. 1959. Separable utility and aggregation. *Econometrica* 27: 469--481.

Hamilton, J., A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, 57, 357–384, 1989.

Hamilton, J., Regime-switching models, *Palgrave Dictionary of Economics*, 2005.

Hjertstrand, Per, J. L. Swofford, and G. Whitney (2016). "Mixed Integer Programming Revealed Preference Tests of Utility Maximization and Weak Separability of Consumption, Leisure, and Money," *Journal of Money, Credit, and Banking* **48**(7): 1547-1561.

Neumann, M. J. M. and C. Greiber (2004), "Inflation and core money growth in the euro area", Bundesbank Discussion Paper no. 36/2004.

Rayton, B. A. and Pavlyk, K., 2010. "On the recent divergence between measures of the money supply in the UK." *Economics Letters*, 108(2):159-162.

Serletis, A. and P. Gogas (2014). "Divisia Monetary Aggregates, the Great Ratios, and Classical Money Demand Functions," *Journal of Money, Credit and Banking* **46**(1): 229-41.

Serletis, A. and S. Rahman (2013). "The Case for Divisia Money Targeting," *Macroeconomic Dynamics* **17**: 1638-1658.

Serletis, A., and Robb. A. L. 1986. "Divisia aggregation and substitutability among monetary assets." *Journal of Money, Credit, and Banking* 18, 430--446

- Serletis, A., Shahmoradi, A., 2005. Semiparametric estimates of the demand for money in the United States. *Macroeconomic Dynamics* 9(4), 542-559.
- Serletis, A., Shahmoradi, A., 2007. Flexible functional forms, curvature conditions, and the demand for assets. *Macroeconomic Dynamics* 11, 455-486.
- Stracca, L. (2004). Does Liquidity Matter? Properties of a Divisia Monetary Aggregate in the Euro Area. *Oxford Bulletin of Economics and Statistics* 66 (3), 309-331.
- Strotz RH. 1957. The empirical implications of a utility tree. *Econometrica* 25: 169--180.
- Strotz RH. 1959. The utility tree: a correction and further appraisal. *Econometrica* 27: 482--488.
- Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.
- Woodford, Michael (2005), "Central Bank Communication and Policy Effectiveness," NBER working paper no. 11898, December.
- Woodford, Michael (2013), "Monetary Policy Targets after the Crisis," presented at the Rethinking Macro Policy II: First Steps and Early Lessons conference, Hosted by the IMF, Washington, DC, April 16-17.